

MATH 135 – Sample Final Exam Review

This review is a collection of sample questions used by instructors of this course at Missouri State University. It contains a sampling of problems representing the material covered throughout the semester and may not contain every type of question on the final exam. Any material listed on the lecture schedule and/or the assignment sheet may be on the final exam. Please also be aware that a few questions on the final exam, while requiring knowledge and understanding of the content covered in the course, may be presented in a form different than the problems in the text.

Short Answer Section

- 1) True or False: $(a+b)^2 = a^2 + b^2$
- 2) True or False: $\frac{4+\sqrt{3}}{2} = 2 + \sqrt{3}$
- 3) True or False: $e^x \cdot e^x = e^{2x}$
- 4) True or False: $e^x + e^x = e^{2x}$
- 5) True or False: $\log_2 x + \log_2 5 = \log_2(5x)$
- 6) True or False: $\ln x - \ln 6 = \frac{\ln x}{\ln 6}$
- 7) What is the relationship of the lines $y = 3x - 2$ and $y = 3x + 5$?
- 8) What do you know about the lines $y = \frac{2}{3}x$ and $y = -\frac{3}{2}x + 1$?
- 9) Can a function have 2 y -intercepts? Explain.
- 10) Give the y -intercept for $y = x^2 - x - 6$.
- 11) Give the x -intercept for $y = x^2 - x - 6$.
- 12) Give the y -intercept for $f(x) = \log_2(x+2)$
- 13) Give the x -intercept for $f(x) = \log_2(x+2)$
- 14) Can a parabola of the form $y = ax^2 + bx + c$ have an inverse function? Why or why not?
- 15) If $f(2) = 5$, $f^{-1}(5) = \underline{\hspace{2cm}}$

16) If the point $(3, -10)$ lies on the graph of $f(x)$, give a point on $f^{-1}(x)$.

17) If $f(x)$ has domain $\{2, 3, 4, 5\}$ and range $\{4, 9, 16, 25\}$, what is the domain of $f^{-1}(x)$?

18) Is $x = 10$ in the domain of $f(x) = \sqrt{3-x}$? Explain.

19) What is the domain of $y = x^2 - 4$?

20) What is the range of $y = x^2 - 4$?

21) Does the graph to the right represent a function?

22) Give the domain of the graph to the right.

23) Give the range of the graph to the right.

24) Sketch any one-to-one function.

25) Sketch any function that is not one-to-one.

26) How does the graph of $y = |x - 3| + 5$ compare to $y = |x|$?

27) What is the minimum value of $f(x) = x^2 + 3$? Explain.

28) Does the graph of $y = \frac{x-8}{x-2}$ cross the line $x = 2$? Explain.

29) What are the asymptotes of $y = \frac{2x-1}{3x-5}$?

30) Does $y = ax^2 + bx + c$ have a maximum or minimum value when $a < 0$?

31) Write a quadratic equation in standard form $(ax^2 + bx + c = 0)$ whose solution set is $\{-2, 7\}$.

32) Write $\log_4 a + \log_4 b$ as a single logarithm.

33) Write an equation to represent the following statement: "y varies jointly as x and z."

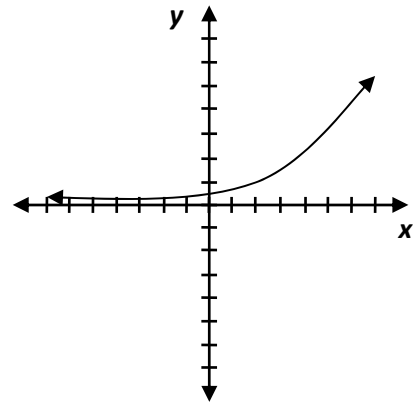
34) Write an equation to represent the following statement: "m varies inversely as n."

35) Evaluate $\log_b \sqrt{b}$

36) Evaluate $\log_a \left(\frac{1}{a}\right)$

37) Evaluate $\log_b 1$

Use this graph to answer 21-23



- 38) What is the relationship of $f(x) = \log_2 x$ and $g(x) = 2^x$?
- 39) Between what two consecutive integers will you find $\log_2 10$?
- 40) Does the model $A(t) = 90e^{-0.1t}$ represent exponential growth or decay? Explain.
- 41) What is an arithmetic sequence?
- 42) What is a geometric sequence?
- 43) Evaluate $\sum_{k=1}^{50} 3$.
- 44) Find the common ratio in the geometric sequence: 1715, 1225, 875, 625
- 45) Write a formula for the general term (a_n) of the sequence 9, 5, 1, -3
- 46) If the sequence $\{1, 4, \dots\}$ satisfies $a_n = 3a_{n-1} + 5a_{n-2}$, then the third term of this sequence is _____.

End of Short Answer Section.

Remaining problems require work to support answers.

Give the domain of each function.

- | | | | |
|-------------------------|------------------------------|--------------------------------|-----------------------|
| 1) $f(x) = \sqrt{3x+2}$ | 2) $h(x) = \frac{x-1}{x-5}$ | 3) $f(x) = \sqrt{3-x}$ | 4) $f(x) = \log(x-8)$ |
| 5) $f(x) = 4x^2 + 5$ | 6) $f(x) = \frac{5x}{x^2-1}$ | 7) $g(x) = \frac{3x+1}{x^2+4}$ | 8) $g(x) = 5x-1$ |

Evaluate.

- | | | | |
|------------------|---------------------------|----------------------------|---------------------------|
| 9) $\log_5 125$ | 10) $\log_9 \frac{1}{81}$ | 11) $\log_8 32$ | 12) $\log(.01)$ |
| 13) $\ln e^{2x}$ | 14) $\log_7 \sqrt[3]{7}$ | 15) $\log_3 36 - \log_3 4$ | 16) $\log_6 4 + \log_6 9$ |

Solve each equation. Leave all answers exact. DO NOT ROUND.

17) $2^{2x+1} = 8$

18) $(x+3)^2 = 25$

19) $x^2 = 2x+1$

20) $x - 13\sqrt{x} + 40 = 0$

21) $(3-8p)^2 = -10(3-8p) - 21$

22) $\log_2 8 - \log_5 x = \log_5 4$

23) $2y^{\frac{2}{5}} - 5y^{\frac{1}{5}} - 3 = 0$

24) $x^2 - 4x + 29 = 0$

25) $\log_2 x = -3$

26) $2^{(5-3x)} = \frac{1}{16}$

27) $\log_x 64 = 3$

28) $\log_x \left(\frac{8}{27} \right) = 3$

29) $\log(5x) = \log 4 + \log(x-3)$

30) $\log_9 x^2 = \log_9 (7x+8)$

31) $\log_3(x+1) - \log_3 4 = 2$

32) $\left(\frac{1}{4} \right)^x = 17$

33) $7^x = 6^{x+7}$

34) $\log x + \log 6 = 2$

35) $5e^{3x-1} = 25$

36) $\log x + \log(x-3) = 1$

Solve each inequality, graph its solution on a number line, then leave the solution in interval notation.

37) $|3x-5|+9 > 2$

38) $|3x-2|+1 \leq 4$

39) $x^2 + 3x \leq 0$

40) $x^2 - 7x > -10$

41) $\frac{3x}{x-2} \geq 4$

42) $\frac{3x+1}{x-7} > 0$

Solve each system of equations.

43) $\begin{cases} x^2 + y^2 = 74 \\ x - y = 2 \end{cases}$

44) $\begin{cases} xy = 20 \\ x + y = 9 \end{cases}$

45) $\begin{cases} x^2 + y^2 = 8 \\ xy = 4 \end{cases}$

46) $\begin{cases} x + y + z = 7 \\ x - y + 2z = 7 \\ 5x + y + z = 11 \end{cases}$

Solve each system of inequalities.

47) $\begin{cases} y \geq x^2 - 3 \\ y < 4 - x \end{cases}$

48) $\begin{cases} 2x - y \leq 4 \\ 3x + 2y > -6 \end{cases}$

49) $\begin{cases} 3x + y \leq 6 \\ x \geq -2 \\ y \leq 4 \end{cases}$

Find the linear equation to represent each of the following conditions.

50) Write the equation of the line through (2, 4) with slope of $-\frac{4}{7}$.

51) Write the equation of the line through the points (3,-7) and (8,-4).

52) Write the equation of the line through (0,1) and perpendicular to $2x - 3y = 5$.

53) Write the equation of a line through (3,1) and parallel to $2x - 3y = 5$.

54) Write the equation of a line parallel to $y + 5 = 2$ and passing through the point (-4,6).

55) Write the equation of a line perpendicular to $y = 8$ and through (-1,-5).

56) Write the equation of a line parallel to $y = 8$ and through (-1,-5).

Find each of the following given $f(x) = x^2 + 1$; $g(x) = 3x - 5$; $h(x) = \begin{cases} 3x & \text{if } x < 0 \\ x - 2 & \text{if } 0 \leq x \leq 10 \\ 5 & \text{if } x > 10 \end{cases}$

57) $h(12)$

58) $h(0)$

59) $g(t+1)$

60) $f(3x)$

61) $(f+h)(-1)$

62) $(f+g)(x)$

63) $(f-g)(x)$

64) $(g \circ f)(2)$

65) $(f \circ g)(x)$

66) $g^{-1}(2)$

Find the inverse for each one-to-one function.

67) $f(x) = 2x - 8$

68) $f(x) = \frac{2x+1}{3}$

69) $f(x) = \frac{3x-2}{2x+1}$

70) $f(x) = \sqrt[5]{x-3}$

Write out the first four terms of each sequence and identify each as Arithmetic, Geometric, or neither. If Arithmetic give the common difference, if Geometric give the common ratio.

71) $a_n = 3n - 5$

72) $a_n = n^2 - n$

73) $\sum_{k=0}^5 \left(\frac{k+1}{k+2} \right)$

74) $a_n = \frac{1}{2}(2)^n$

75) $\sum_{k=1}^{10} \left(\frac{2k+1}{2} \right)$

Find the n th term, a_n , of each sequence.

76) $1, \frac{7}{4}, \frac{5}{2}, \frac{13}{4}, \dots$

77) $-\frac{27}{20}, \frac{9}{10}, -\frac{3}{5}, \frac{2}{5}, \dots$

78) an arithmetic sequence with 10th term 16; 15th term -29

79) Use the appropriate formula(s) to find a_{80} for the arithmetic sequence where $a_1 = -4$, $d = 4$.

80) Use the appropriate formula(s) to find the sum of the first 10 terms: -6, -1, 4, 9, ...

81) Use the appropriate formula(s) to find the sum: $43 + 47 + 51 + 55 + \dots + 403$

Use the appropriate formula to evaluate.

82) $\sum_{k=1}^{16} (2k + 7)$

83) $\sum_{i=1}^5 3(-2)^i$

84) $\sum_{k=1}^{\infty} 5(2)^k$

85) $\sum_{i=1}^{\infty} 2\left(\frac{1}{5}\right)^i$

86) Use the binomial theorem to expand the expression: $(2x - 4)^5$

87) Find the 5th term in the expansion of $(x - y)^{11}$

88) Find the 10th term in the expansion of $(3x + 2)^{12}$

89) Find the term containing x^4 in the expansion of $(2x - 1)^7$

Write the equation that relates the quantities given. DO NOT SOLVE.

90) The illumination, I , produced on a surface by a source of light varies directly as the candlepower, c , of the source and inversely as the square of the distance, d , between the source and the surface.

91) Express the area, A , of a rectangle as a function of the width, x , if the length is twice the width of the rectangle.

92) A commissioned sales person earns \$100 base pay plus \$10 per item sold. Express her gross salary, G , as a function of the number of items sold, x .

Solve each problem. Remember to identify variables and give the equation used to solve each problem.

- 93) A baseball diamond is actually a square with 90-foot sides.
What is the distance from home plate to second base?
- 94) A pool measuring 10 meters by 20 meters is surrounded by a walkway of uniform width. If the area of the pool and the walkway combined is 600 square meters, what is the width of the walkway?
- 95) The illumination provided by a car's headlight varies inversely as the square of the distance from the headlight. A car's headlight produces an illumination of 3.75 footcandles at a distance of 40 feet. What is the illumination when the distance is 50 feet?
- 96) The net income, y , (given in millions of dollars) of Pet Products Unlimited from 2004 to 2006 is given by the equation $y = 9x^2 + 15x + 52$, where x represents the number of years after 2004. Assume this trend continues and predict the year in which Pet Products Unlimited's net income will be \$598 million.
- 97) Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p dollars, the revenue R (in dollars) is $R(p) = -4p^2 + 4000p$. a) What unit price should be established for the dryer to maximize revenue? b) What is the maximum revenue?
- 98) Since 1950, the growth in the world population in millions closely fits the exponential function $A(t) = 2600e^{0.018t}$, where t is the number of years since 1950. Estimate the population in the year 2018 to the nearest million.
- 99) The formula $D = 6e^{-0.04h}$ can be used to find the number of milligrams D of a certain drug in a patient's bloodstream h hours after the drug has been given. When the number of milligrams reaches 2, the drug is given again. What is the time (in hours) between injections?
- 100) If Emery has \$1800 to invest at 6% per year compounded monthly, how long will it be before he has \$2700? If the compounding is continuous, how long will it be? (round your answers to 3 decimal places)
- 101) Cindy will require \$20,000 in 4 years to return to college to get an MBA degree. How much money should she ask her parents for now so that, if she invests it at 9% compounded continuously, she will have enough for school?
- 102) How long does it take \$1700 to double if it is invested at 5% interest, compounded quarterly?
- 103) The half-life of silicon-32 is 710 years. If 100 grams is present now, how much will be present in 600 years? (Use the exponential growth/decay formula, round answer to 3 decimal places.)
- 104) A person puts \$20 into a bank account on January 1, \$30 on February 1, \$40 on March 1, and so forth. How much has the person put into the bank account by December 30?
- 105) A machine depreciates by 25% of its value each year after purchase. If it cost \$42,000 new, what is its value after 7 years?

- 106) Two kinds of crated cargo, A and B, are to be shipped by truck. The weight and volume of each type are given in the following table:

	Cargo A	Cargo B
Volume	50 cubic feet	10 cubic feet
Weight	200 pounds	360 pounds

- The shipping company charges \$75 per crate for cargo A and \$100 per crate for cargo B. The truck has a maximum load limit of 7200 pounds and 1000 cubic feet. How many of each type of cargo should be shipped to maximize profit for the shipping company? What is the maximum possible profit?
- 107) A manufacturer produces two models of mountain bicycles. Model A requires 5 hours for assembling and 2 hours for painting. Model B requires 4 hours for assembling and 3 hours for painting. The assembly department has 200 hours available for the bikes. The paint department has 108 hours available. The profits are \$25 per model A bike and \$15 per model B bike. How many of each type of mountain bicycle should be produced to maximize profit?

Graph each of the following functions. Label reference points and all asymptotes.

108) $5y = 10 - 3x$

113) $y = 3x^2 + 6x + 6$

118) $f(x) = 2^{x+1}$

109) $y = \frac{2x+9}{3}$

114) $f(x) = (x-2)^3(x+3)^2(x-4)$

119) $f(x) = \log_2(x) + 3$

110) $y = -2x^2 + 5$

115) $f(x) = 2 - x^3$

120) $y = 2 - \ln(x+4)$

111) $f(x) = -\frac{5}{2-3x}$

116) $f(x) = \frac{5x}{(x-1)(x+4)}$

121) $f(x) = \frac{(x+3)(x+1)}{x^2 + 8x + 16}$

112) $f(x) = e^{-x} - 1$

117) $f(x) = \log_3(x-2)$

122) $f(x) = \begin{cases} \frac{2}{3}x & \text{if } x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$

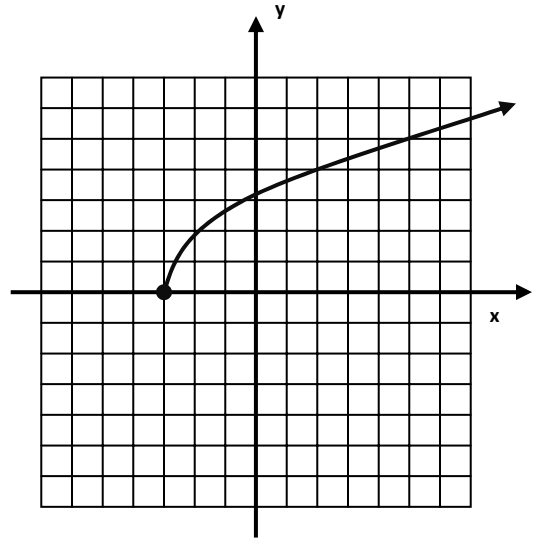
Sketch each of the following, given the graph of $f(x)$ to the right.

123) $f^{-1}(x)$

124) $y = 2f(x) - 3$

125) $y = -f(x+5)$

126) $g(x) = f(x-2) + 1$



Solutions

SHORT ANSWER SECTION

- 1) False
- 2) False
- 3) True
- 4) False
- 5) True
- 6) False
- 7) They are parallel
- 8) They are perpendicular
- 9) No, explanation required
- 10) $(0, -6)$
- 11) $(3, 0)$ and $(-2, 0)$
- 12) $(0, 1)$
- 13) $(-1, 0)$
- 14) No, explanation required
- 15) $f^{-1}(5) = 2$
- 16) $(-10, 3)$
- 17) $\{4, 9, 16, 25\}$
- 18) No, explanation required
- 19) $(-\infty, \infty)$
- 20) $[-4, \infty)$
- 21) Yes
- 22) $(-\infty, \infty)$
- 23) $(0, \infty)$
- 24) Answers will vary
- 25) Answers will vary
- 26) It is shifted up 5 units and to the right 3 units
- 27) Min = 3, explanation required
- 28) No, explanation required
- 29) HA: $y = \frac{2}{3}$; VA: $x = \frac{5}{3}$
- 30) Maximum
- 31) $x^2 - 5x - 14 = 0$

32) $\log_4(ab)$

33) $y = kxz$

34) $m = \frac{k}{n}$

35) $\frac{1}{2}$

36) -1

37) 0

38) They are inverse functions

39) 3 and 4

40) Decay, explanation required

41) Answers will vary

42) Answers will vary

43) 150

44) $r = \frac{5}{7}$

45) $a_n = -4n + 13$

46) $a_3 = 17$

REMAINDER OF THE REVIEW REQUIRES WORK TO SUPPORT ANSWERS

1) $\left[-\frac{2}{3}, \infty\right)$

2) All real numbers except 5

3) $(-\infty, 3]$

4) $(8, \infty)$

5) $(-\infty, \infty)$

6) All reals except ± 1

7) $(-\infty, \infty)$

8) $(-\infty, \infty)$

9) 3

10) -2

11) $\frac{5}{3}$

12) -2

13) $2x$

14) $\frac{1}{3}$

15) 2

16) 2

17) $x = 1$

18) $x = -8, 2$

19) $x = 1 \pm \sqrt{2}$

20) $x = 25, 64$

21) $x = \frac{3}{4}, \frac{5}{4}$

22) $x = \frac{125}{4} = 31.25$

23) $y = -\frac{1}{32}, 243$

24) $x = 2 \pm 5i$

25) $x = \frac{1}{8}$

26) $x = 3$

27) $x = 4$

28) $x = \frac{2}{3}$

29) \emptyset

30) $x = -1, 8$

31) $x = 35$

32) $x = \frac{\ln 17}{\ln(.25)} = \frac{\log 17}{\log(.25)}$

33) $x = \frac{7 \ln 6}{\ln 7 - \ln 6} = \frac{7 \ln 6}{\ln\left(\frac{7}{6}\right)}$
 $= \frac{7 \log 6}{\log 7 - \log 6} = \frac{7 \log 6}{\log\left(\frac{7}{6}\right)}$

34) $x = \frac{50}{3}$

35) $x = \frac{1 + \ln 5}{3}$

36) $x = 5$

37) $(-\infty, \infty)$

38) $\left[-\frac{1}{3}, \frac{5}{3}\right]$

39) $[-3, 0]$

40) $(-\infty, 2) \cup (5, \infty)$

41) $(2, 8]$

42) $\left(-\infty, -\frac{1}{3}\right) \cup (7, \infty)$

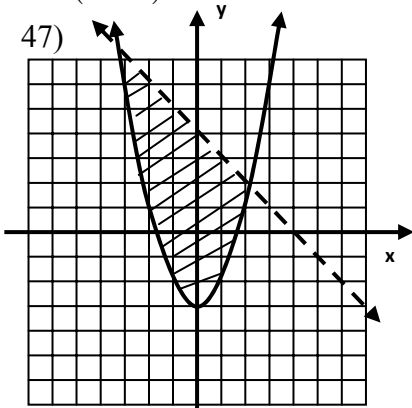
43) $(-5, -7) ; (7, 5)$

44) $(5, 4) ; (4, 5)$

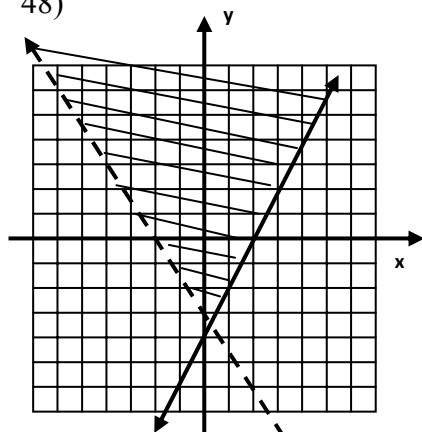
45) $(2, 2) ; (-2, -2)$

46) $(1, 2, 4)$

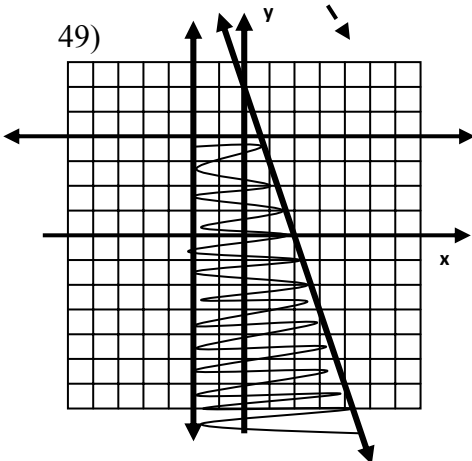
47)



48)



49)



50) $y = -\frac{4}{7}x + \frac{36}{7}$

51) $y = \frac{3}{5}x - \frac{44}{5}$

52) $y = -\frac{3}{2}x + 1$

53) $y = \frac{2}{3}x - 1$

54) $y = 6$

55) $x = -1$

56) $y = -5$

57) 5

58) -2

59) $3t - 2$

60) $9x^2 + 1$

61) -1

62) $x^2 + 3x - 4$

63) $x^2 - 3x + 6$

64) 10

65) $9x^2 - 30x + 26$

66) $\frac{7}{3}$

67) $f^{-1}(x) = \frac{x+8}{2} = \frac{1}{2}x + 4$

68) $f^{-1}(x) = \frac{3}{2}x - \frac{1}{2} = \frac{3x-1}{2}$

69) $f^{-1}(x) = \frac{-x-2}{2x-3} = \frac{x+2}{3-2x}$

70) $f^{-1}(x) = x^5 + 3$

71) -2, 1, 4, 7; Arith; d=3

72) 0, 2, 6, 12; Neither

73) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$; Neither

74) 1, 2, 4, 8; Geom; r=2

75) $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$; Arith; d=1

76) $a_n = \frac{3}{4}n + \frac{1}{4}$

77) $a_n = -\frac{27}{20} \left(-\frac{2}{3}\right)^{n-1}$

78) $a_n = 106 - 9n$

79) $a_{80} = 312$

80) $S_{10} = 165$

81) 20,293

82) $S_{16} = 384$

83) $S_5 = -66$

84) Inf sum does not exist

85) $S_\infty = \frac{1}{2}$

86) $32x^5 - 320x^4 + 1280x^3 - 2560x^2 + 2560x - 1024$

87) $-330x^7 y^4$

88) $220(27x^3)(512) = 3,041,280x^3$

89) $-560x^4$

90) $I = \frac{kc}{d^2}$

91) $A(x) = 2x^2$

92) $G(x) = 100 + 10x$

93) $\sqrt{16200} \approx 127.3$ feet

94) 5 meters

95) 2.4 footcandles

96) After about 7.5 years; in the year 2011

97) a) \$500 b) \$1,000,000

98) 8842 million

99) About 27.5 hours

100) Monthly about 6.775 yrs Continuously ≈ 6.758 yrs

101) About \$13,954

102) Almost 14 years

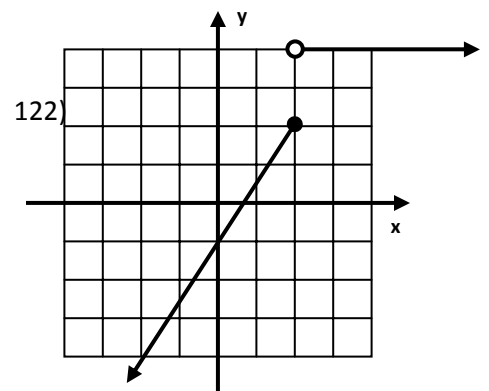
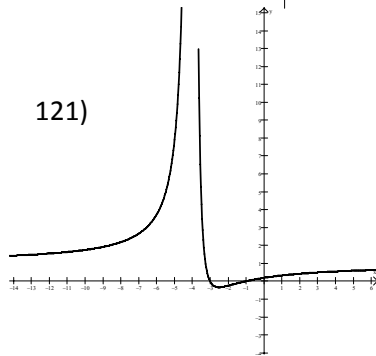
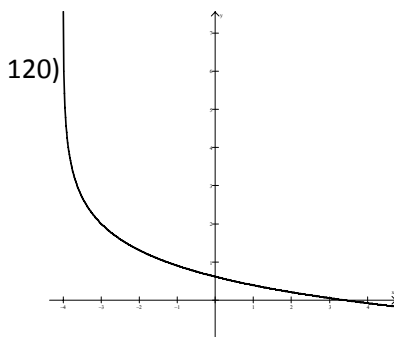
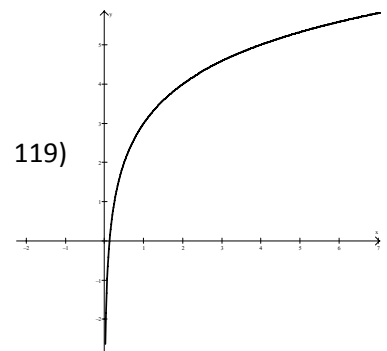
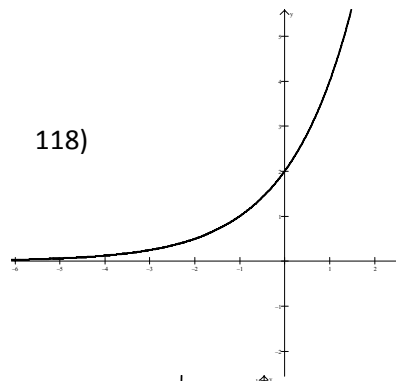
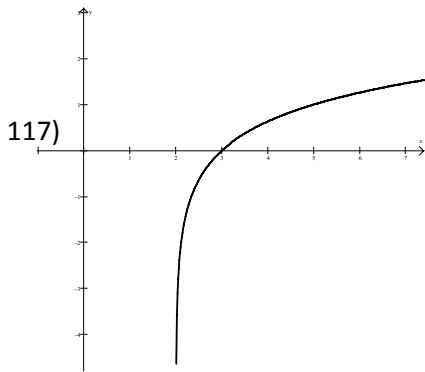
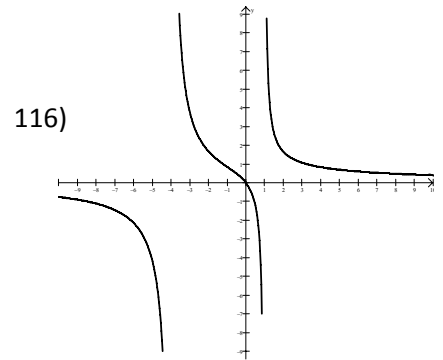
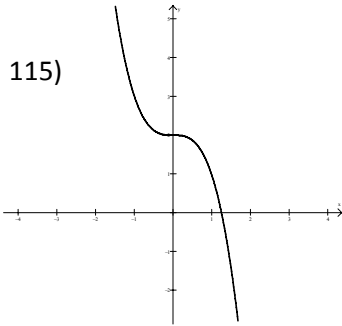
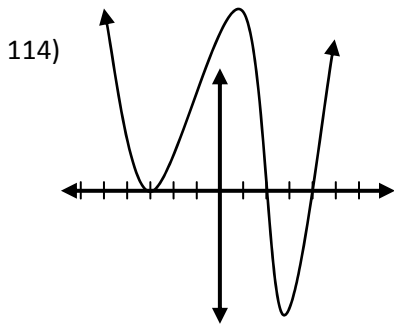
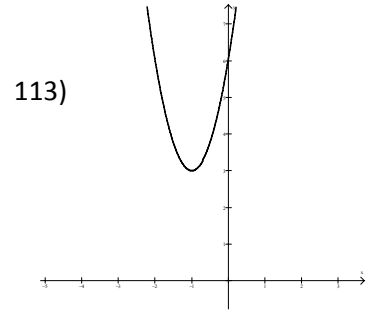
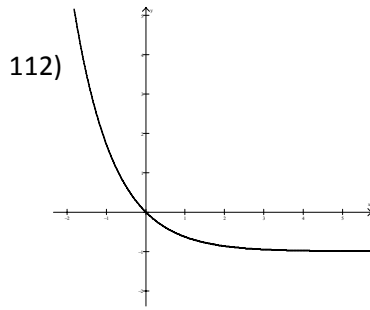
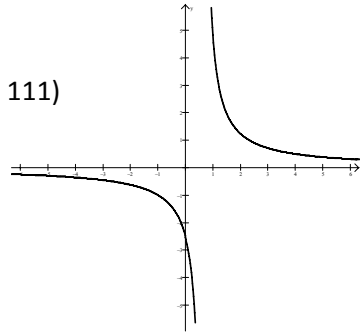
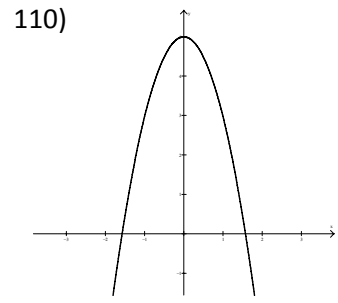
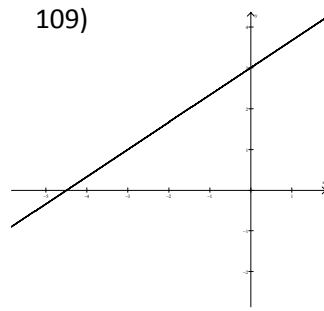
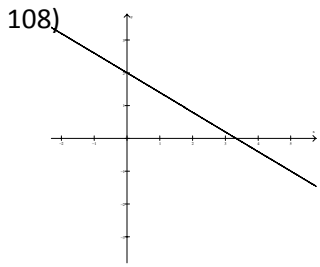
103) About 55.668 grams

104) \$900

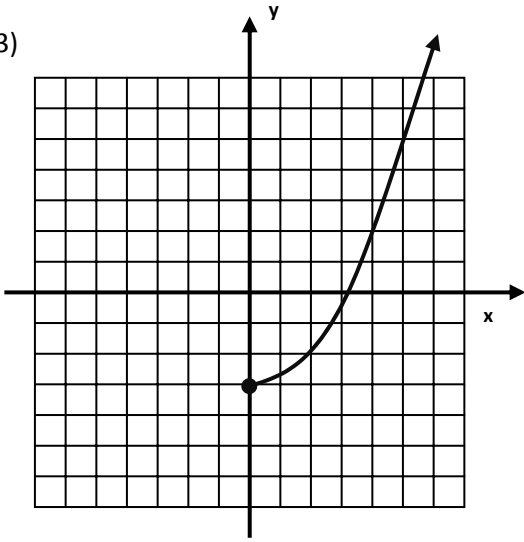
105) About \$5606

106) Max Profit: \$2350
18 crates of cargo A
and 10 crates of cargo B

107) 40 model A bikes and no model B bikes

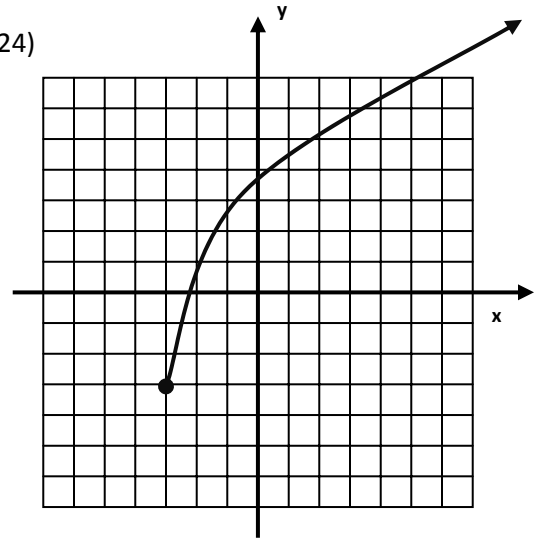


123)



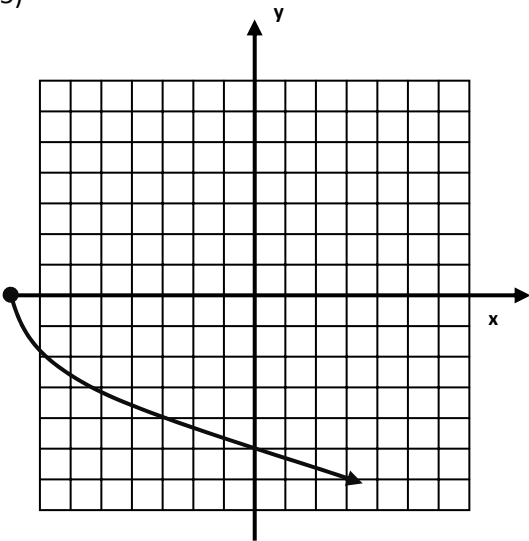
$$f^{-1}(x)$$

124)



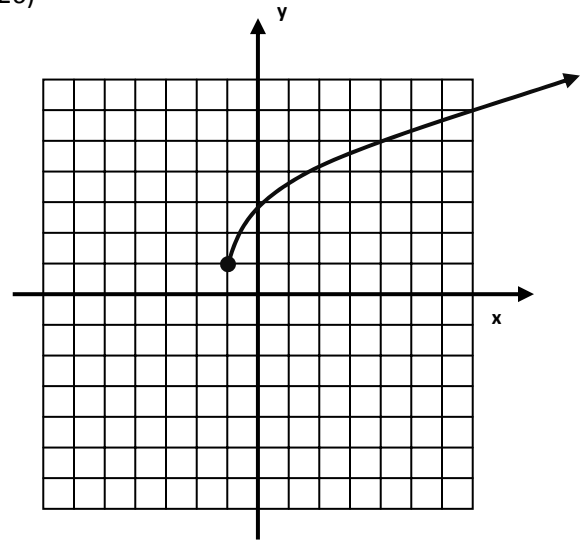
$$y = 2f(x) - 3$$

125)



$$y = -f(x+5)$$

126)



$$g(x) = f(x-2) + 1$$