

To solve an polynomial inequality

$$(\dots)(\dots) \dots (\dots) > 0$$

or

$$(\dots)(\dots) \dots (\dots) < 0$$

or

$$(\dots)(\dots) \dots (\dots) \geq 0$$

or

$$(\dots)(\dots) \dots (\dots) \leq 0$$

- Note that all the polynomial's zeros (or roots) will divide the real number line into several regions (intervals), and
- The polynomial  $(\dots)(\dots) \dots (\dots)$  will either completely positive or completely negative in each region (interval). This is because the polynomial can change its algebraic sign without going through any of its zeros (roots).
- Therefore, to solve  $(\dots)(\dots) \dots (\dots) > 0$ , we only have to find out in which region the polynomial is positive. Similarly, to solve  $(\dots)(\dots) \dots (\dots) < 0$ , we only have to find out in which region the polynomial is negative. For the other two cases, we only have to include the all the zeros.
- Since the polynomial will only have one algebraic sign in each region, one can use an arbitrary test point from this region to determine the sign, and thus build a complete sign chart for this polynomial before actually determining the solution.
- A more efficient way is to understand that the sign in the rightmost region will have the same sign as the leading coefficient of this polynomial.
- Next, if all factors  $(\dots)$  of this polynomial are distinct, the signs in the sign chart will alternate. This is because the polynomial will change sign each time it passes through a zero.

In addition,

- One can extend the alternating feature to all cases when the exponents of all factors are odd!
- Also, the sign will not change around a zero (or root) of the polynomial if the factor producing this zero (or root) has an even exponent!