Diffusive Predator-Prey Models with Nonconstant Harvesting

Mathematical models of competition, mutualism and predator-prey are among the most studied problems in population dynamics for multiple species. In general, mathematical models for these problems can be expressed as a set of parameterized difference or differential equations, or dynamical systems.

Extensive research has been going on in this direction. In [2] and [9], predator-prey models with harvesting activity are studied, some numerical examples are provided, and rich dynamics are observed. The inclusion of motion of the species in this kind of models is considered in [1], which leads to reaction-diffusion models.
The author proposes to study the qualitative behavior of solutions of continuous predator-prey models, with harvesting. A theoretical and numerical bifurcation analysis of such problems should reveal a rich dynamics underlying these equations. The student will work on finding domains of values of the parameters for which special solutions and properties are observed, including steady states, periodic orbits, bifurcations, stability and chaos. Interpretations of these results will be given with concrete examples. Although most literature focuses on constant harvesting, we are interested in the study of asymptotic behavior of solutions and the effects of nonconstant harvesting. The student will provide a sound theoretical explanation of these phenomena and will illustrate the main ideas with several numerical examples. Software and numerical algorithms from [3, 5] will be essential for these goals.

Ratio-dependent models with constant harvesting are studied in [10]. As an extension, some results for ratio-dependent models with nonconstant harvesting have been obtained in [6]. Taking this work as starting point, there are several topics that can be approached. The first one would be the inclusion of a diffusion term into the system to account for the motion of the species. This will result in a set of partial differential equations similar to those in [1]. This in turn will lead to the study of propagating fronts for the reaction-diffusion system, which could be understood as separatrices on the phase space as discussed in [4, 7]. Different harvesting functions can be studied, on both, the predator and the prey, to complement the study of harvesting of the prey in [6] and [10]. Ruan et al. [8] show that some harvesting rate has a stabilizing effect, and also give a discussion on the maximum sustainable yields. These topics are mathematically challenging, and will also be considered for the continuation of the REU program in the next years.

Prerequisites: A basic background on linear algebra and differential equations. Optimally, some programming experience is desirable, but not required.

References

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