

Hurwitz Groups

Hurwitz groups are finite groups that can be generated by two elements x and y subject to the relations $x^2 = y^3 = (xy)^7 = 1$. They are certain automorphism groups of Riemann surfaces [1]. Although some information is known (for example which alternating groups are Hurwitz and the fact that $SL_n(\mathbb{F}_q)$ is Hurwitz for all sufficiently large n), there are still a vast number of questions for an undergraduate to investigate.

A related avenue to explore is the determination of which finite groups are generated by two elements x and y subject to the relations $x^a = y^b = (xy)^c = 1$ with $1/a + 1/b + 1/c < 1$.

The “hands-on” nature of this project along with its long pedigree should attract undergraduates. A student involved in this project should come away with an increased understanding of the symmetric group, the classical groups over finite fields, and representation theory.

Prerequisites: One semester of abstract algebra.

References

[1] M. Conder, *Hurwitz groups, a brief survey*, Bulletin AMS **23** (1990), 359-370.