

Properties of the Subgroup Graph of a Group

Under the author's supervision, Joseph P. Bohanon wrote a master's thesis [1] in which he analyzed the planarity of the subgroup (or Hasse) graph of a group. Using a variety of techniques and results (including the classification of minimal simple groups), he was able to completely characterize the finite groups whose subgroup graph is planar. Bohanon and the author are currently investigating groups having an Eulerian subgroup graph [2] and have completely characterized the finite abelian groups and symmetric groups with this property. Unlike the planarity question, we do not believe that a complete classification of Eulerian groups is likely. This leaves a number of families of groups (dihedral, other semi-direct products, linear groups, p -groups, etc.) for investigation by undergraduates.

Other graph-theoretical properties of the subgroup graph that might be studied include path-Eulerian, Hamiltonian, path-Hamiltonian, and the chromatic number. Although it would be more difficult, generalizing Bohanon's work to the problem of determining the genus (oriented or non-oriented) of the subgroup graph (at least for a restricted family of groups), would also be of interest.

Finally, the subgroup graph is also a lattice. Determining the Möbius function of that lattice is critical in determining the probability that k distinct elements generate a group. In her thesis [3], Christie Tosh Bowerman has addressed this problem for a number of groups, but there are many families of groups left to investigate.

In the author's experience, the synergism between the group-theoretical and graph-theoretical aspects of these problems makes this project particularly appealing to undergraduates. Upon completion of the project, participants should have a greatly deepened understanding of both group theory and graph theory.

Prerequisites: One semester of abstract algebra. Some exposure to graph theory is desirable, but not required.

References

[1] J.P. Bohanon and L. Reid, *Finite groups with planar subgroup lattices*, Journal of Algebraic Combinatorics, to appear.

[2] J.P. Bohanon and L. Reid, *Families of finite groups with Eulerian subgroup lattices*, in progress.

[3] C.Tosh Bowerman, *Probabilistic Questions in Group Theory*, Master's Thesis, Missouri State University, in progress.