Abstracts

Invited Talk: "Knot Theory 101"
Dr. Mark Rogers, Missouri State University

This talk will be a basic introduction to knot theory, accessible to a general audience. I will introduce the concept of knots, discuss a few of the invariants used to distinguish knots, and look at a few open questions. Time permitting, I will introduce rational tangles and explain their correspondence with rational numbers.

"The Buffered Fourier Spectral Method"
Monica Davanzo, University of Central Arkansas
Faculty Advisor: Dr. Yinlin Dong, University of Central Arkansas

The standard Fourier spectral method is efficient for solving problems with periodic boundary conditions, but oscillations occur for problems with non-periodic boundary conditions. This can be corrected using a buffered Fourier spectral method. For non-periodic functions, a buffering polynomial will be added to the right end boundary, making it smooth and periodic on the boundaries, before applying the Fast Fourier Transform. Then the buffering zone can be removed to compute maximum error and order of accuracy. Using this method, the derivatives of non-periodic functions can be approximated and the solutions of select ordinary differential equations can be calculated and the error reduced from 10^{-5} to 10^{-11}.

"Intersections of Shortest Taxicab Paths in the Sierpiński Carpet"
Rebekah Chase, Evangel University
Faculty Advisor: Dr. Carl Hammarsten, Lafayette College

In recent work, Berkove and Smith developed an algorithm to construct shortest taxicab paths in the Sierpiński carpet and some of its higher-dimensional generalizations. We consider an extension of this problem examining minimal area surfaces bound by shortest taxicab paths in higher-dimensional fractals. Such a minimal surface will have zero area if and only if the associated shortest paths have non-empty common intersection. Specifically, we give a set of necessary and sufficient conditions on the relative positions for three points in the carpet which characterize when the pairwise shortest taxicab paths have non-empty triple intersection. Finally, we indicate how our work might generalize to higher dimensions.

“The Numbers Behind Nurikabe”

Jaired Collins, Missouri Southern State University
Jonas Smith, Missouri Southern State University
Faculty Advisors: Dr. Charles Curtis and Dr. Jacob Boswell, Missouri Southern State University

Nurikabe is a logic puzzle, named for an invisible wall in Japanese folklore. It is played in an $m \times n$ grid of cells, with some containing positive integers. The goal is to create a solution of islands and streams that adhere to the following rules: each cell with a number is an island, and the island must contain as many cells as its number; each island may only contain one numbered cell; and there must only be one contiguous stream without any 2×2 pools. We generate all pictures of size $n \times n$, $n = 1$ through 6, that satisfy the rules of Nurikabe. To avoid counting separate puzzles that are fundamentally the same, we group the puzzles into equivalence classes by symmetry, and count each equivalence class as one solution.

“Midpoint Iterations of the Unit Square”

Joe Foster, Washington University
Adam Somers, Missouri State University
Faculty Advisor: Dr, Xingping Sun, Missouri State University

We present findings on a midpoint iterative function system within the unit square, which generates a sequence of convex polygons converging to a convex attractor set of positive Lebesgue measure. We show that the boundary of this attractor set is a compact, smooth 1-manifold, yet fractal-like. Notably, we find that the boundary has a first derivative that is a new variant of the Cantor ternary function (the Devil’s Staircase). The novelty here is reflected in the construction of the underlying Cantor set, which is reminiscent of the way Stern constructed rational numbers. Precisely, we build an infinite binary tree using slopes of the convex polygons in order of the iteration, and prove that it is isomorphic to the Stern-Brocot tree. We reconcile the issue of a smooth 1-manifold being “fractal-like” by adopting Falconer’s viewpoint on the characteristics of fractals, which offers a broader perspective for understanding fractals than Mandelbrot’s original definition.

“Perimeters of Sections of a 3-Dimensional Box”

Wyatt Gregory, University of Missouri, Columbia
Faculty Advisor: Dr. Alexander Koldobsky, University of Missouri, Columbia

We examine the surface content of a central cross-section of an *n*-dimensional box with side lengths $0 \le b_1 \le ... \le b_n$. In particular, if $n = 3$, the maximum value of this perimeter is found to be $2b_3 + 2\sqrt{b_1^2 + b_2^2}$. We obtain these results using Fourier analysis methods that may possibly be generalized to arbitrary convex bodies in *n* dimensions. Questions involving various measurements of cross-sections of convex bodies such as the cube and box are surprisingly difficult. It is hoped that these results will lead to a greater understanding of *n*-dimensional convex bodies.

“Group Strategy Implementation in Multi-Agent Systems”

Aubrey Hormel, Missouri State University
Faculty Advisor: Dr. Jingjin Yu, Rutgers University

A system of multiple agents (e.g. a group of robots or drones) may be assigned a collective task which involves travelling as a group to some target formation. A formation control algorithm published by Yu and LaValle maps and schedules agent paths in such a way that the total distance travelled by the group is minimized and the time required for agents to reach the target formation is bounded. This optimization is made simpler by allowing the algorithm to assign each agent to an optimal position in the target formation (as opposed to predetermined goal positions for each agent). I use this algorithm to model collective strategies for a group of agents, including patrolling a perimeter, surrounding one or more targets, or any combination of these. With enough data gathered from these and similar group tasks, one may begin to train a decentralized, autonomous multi-agent system for practical applications.

“Direction Sets and Maximal Directional Hilbert Transforms”

Caleb Marshall, Missouri State University
Faculty Advisors: Dr. William Bray, Missouri State University and Dr. Malabika Pramanik, University of British Columbia

We begin by discussing Cauchy's theory of integration, which predated the Riemann Integral. We then examine how this method of integration allows us to evaluate certain singular integrals, many of which are familiar to us from Calculus, such as

$$\int_{-\infty}^{\infty} \frac{1}{x}\,dx$$

We then examine how this concept of integration leads to the development of the classical Hilbert Transform, a beautiful singular integral operator that joins Complex and Real Variable integration theories.

Moreover, for a finite set of directions Ω defined on the unit circle, we define a Directional Hilbert Transform for $u \in \Omega$ by

$$\mathcal{H}_u f\left(y_1, y_2\right) = \text{p.v.}\int_{-\infty}^{\infty} f\left(y_1 - t, y_2 - ut\right)\frac{dt}{t}$$

For the associated maximal operator

$$\mathcal{H}_\Omega f\left(y_1, y_2\right) = \max_{u \in \Omega} \mathcal{H}_u\left(\left| f\left(y_1, y_2\right)\right|\right)$$

we give a brief outline of our proof that the lower bound on the operator norm is

$$\frac{\left\|\mathcal{H}_\Omega f\right\|_{L^2}}{\left\| f\right\|_{L^2}} \geq c\log\left(\#\Omega\right)$$

along direction sets with certain geometries, such as “clustered” and Cantor-like direction sets.

"Gaps in Primes of Various Number Rings"

Jake Miles, Missouri State University
Faculty Advisor: Dr. Les Reid, Missouri State University

It has long been known that there exist arbitrarily large gaps between primes in the integers. In 1998, Gethner, Wagon, and Wick examined prime gaps in the ring of Gaussian integers, i.e. $\mathbb{Z}[i]=\{a+bi \mid a,b\in\mathbb{Z}\}$. In particular they examine the question of whether or not there exists a finite distance such that one could "walk to infinity" on the primes of $\mathbb{Z}[i]$ in the complex plane, using step sizes of that distance or less. In this talk we study prime gaps in $\mathbb{Z}[\sqrt{2}]=\{a+b\sqrt{2} \mid a,b\in\mathbb{Z}\}$. We examine these gaps from two perspectives: consider $\mathbb{Z}[\sqrt{2}]$ as a subset of $\mathbb{R}$, or consider it as the set of all ordered pairs (a,b) in the plane. In the former perspective we conjecture that the primes are dense; in the latter we conjecture that we can walk to infinity on the primes with a bounded step size. If time allows, we will also discuss the prime gaps in other number rings.

"Divergent Thinking or Problem Posing: Creativity at its Best"

Demitrius Moore, University of Central Arkansas
Faculty Advisor, Dr. James Fetterly, University of Central Arkansas

Historically, a long-standing connection exists between creativity and problem posing. One way to understand creativity is through divergent thinking. It has been noted in the past that one of the most efficient and effective ways to foster creativity is to be exposed to creativity. This study desires to understand if mathematical exposures and experiences with problem posing and/or divergent thinking affect mathematical creativity in the classroom. By using a sample population of Algebra students, this study seeks to answer which treatment will enhance mathematical creativity, if any. For this study, three treatment groups are considered. The first treatment exposes students to mathematical problem posing activities, the second treatment explores divergent thinking in mathematics, and the third combines both experiences of problem posing and divergent thinking. Over a six-week period, three problem-posing treatments will be administered every other week and, on alternate weeks, three divergent thinking treatment will administered, where the duration of each treatment is one 50-minute class. The collection of pre- and post-test data will test for significant differences in mathematical creativity, beliefs, and knowledge among the three groups to discover which treatment is effective.

"Coloring Knots with Groups"

Gabriel Wallace, Missouri State University
Faculty Advisor: Dr. Mark Rogers, Missouri State University

If G is a group, then we say a knot K is G-colorable if there exists a surjective homomorphism from the knot group onto G. In this talk, we explore what it means for a knot to be G-colorable and a more intuitive method to color a knot with a group. Kenneth Perko in 1975 proved that a knot is S_4-colorable if and only if it is S_3-colorable. Most of his paper is concerned with proving the reverse direction, but uses techniques beyond the scope of this talk. Instead, we focus on the first sentence of Perko's paper proving that S_4-colorable implies S_3-colorable "follows trivially" and unpack his one line proof.

Problem Posing Project: A Serious Fibonacci Fiasco"

Jonathan Zluticky, University of Central Arkansas
Faculty Advisor, Dr. James Fetterly, University of Central Arkansas

In most math classes, both in high school and college, proofs are disconnected from conjecture. The pattern of teaching tends to be: definition, theorem, proof. This misses the development of a skill of an important skill shared by great mathematicians and distort the vision of how new mathematics is formed. "In creating mathematics, problems are posed, examples analyzed, conjectures made, counterexamples offered, conjectured revised; a theorem results when this refinement and validation of ideas answers a significant question" (Battista, Clements 1995). The Reasoning and Proof standards from the NCTM suggests that students should "examine patterns and structures to detect regularities; formulate generalizations and conjectures about observed regularities; evaluate conjectures; and construct and evaluate mathematical arguments" (NCTM 2000). This project shows one method of fostering these skills in students through problem posing and asking the question: "What if not?" It looks at the specific example of $\sum_{n=1}^{\infty}\frac{1}{5^n}$ and begins to change certain aspects of it, identify patterns, make conjectures and generalizations, and then formalize a proof.