Dynamics of a Ratio-Dependent Predator-Prey Model with Nonconstant Harvesting Policies

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Predator-Prey Models

- 1925 & 1926: Lotka and Volterra independently propose a pair of differential equations that model the relationship between a single predator and a single prey in a given environment:

$$\dot{x} = rx - axy$$

$$\dot{y} = bxy - cy$$

Variable and Parameter definitions

x – prey species population

y – predator species population

r – Intrinsic rate of prey population Increase

a – Predation coefficient

b – Reproduction rate per 1 prey eaten

c – Predator mortality rate

Ratio-Dependent Predator-Prey Model

Prey growth term

Predation term

$$\dot{x} = x(1 - x) - \frac{axy}{y + x}$$

$$\dot{y} = -dy + \frac{bxy}{y + x}$$

Predator death term

Predator growth term

Parameter/Variable Definitions

x – prey population

y – predator population

a – capture rate of prey

d – natural death rate of predator

b – predator conversion rate

Previous Research

Harvesting on the Prey Species

$$\dot{x} = x(1 - x) - \frac{axy}{y + x} - h$$

$$\dot{y} = -dy + \frac{bxy}{y + x}$$

First Goal

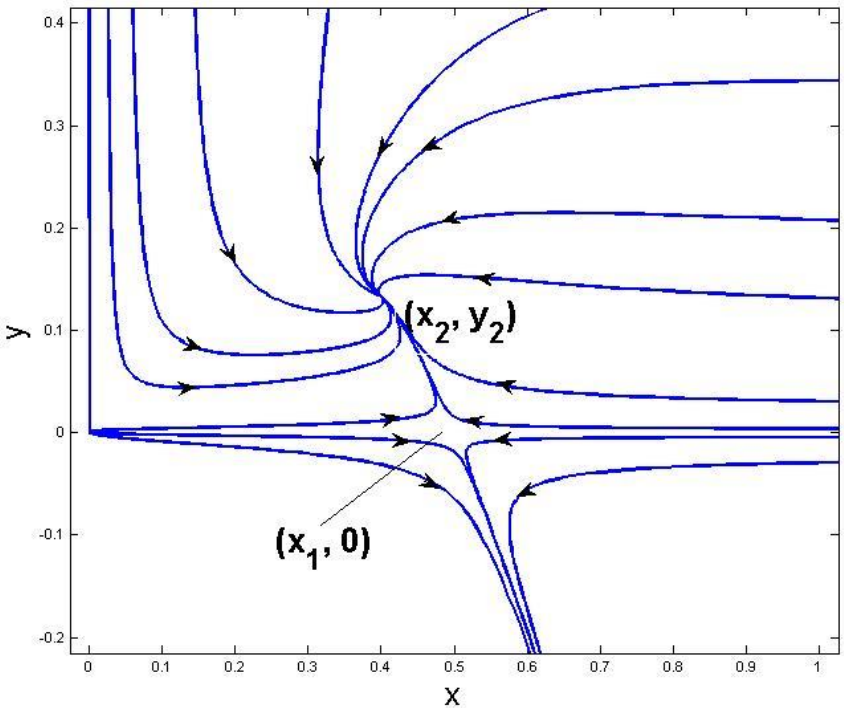
Analyze the model with two non-constant harvesting functions in the prey equation.

1. $H(x) = hx$

2. $H(x) = \dfrac{hx}{c+x}$

Second Goal

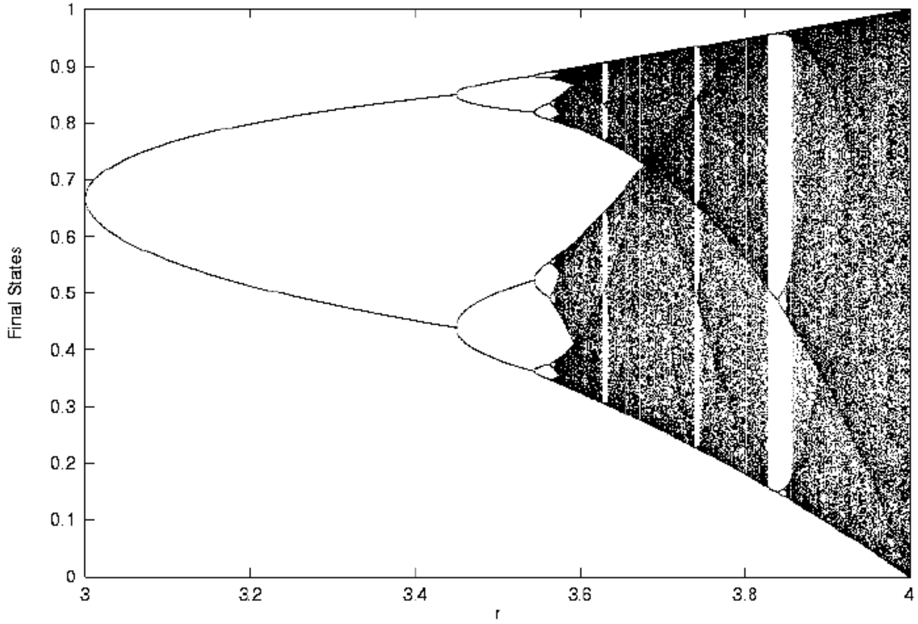
Find equilibrium points and determine local stability.



Third Goal

Find bifurcations, periodic orbits, and connecting orbits.

Logistic Equation Bifurcation Diagram



[Example of Hopf Bifurcation]

Model One: Constant Effort Harvesting

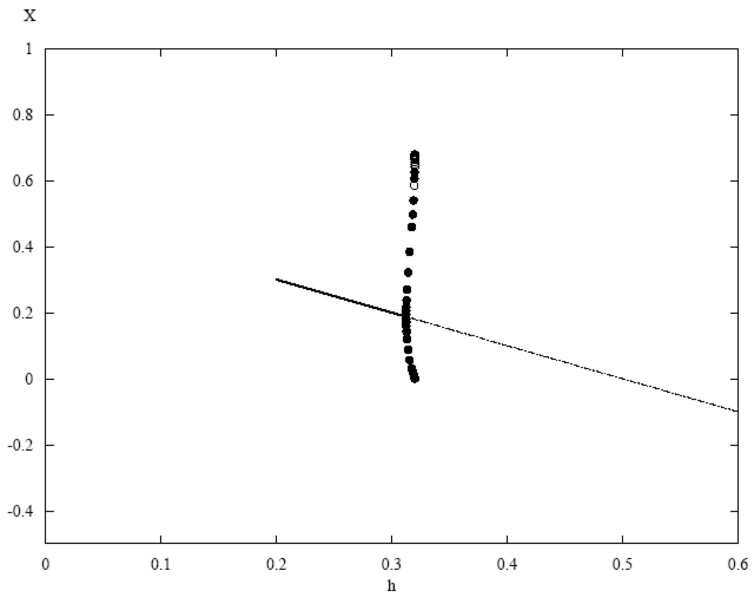
$$\dot{x} = x(1-x) - \frac{axy}{y+x} - hx$$

$$\dot{y} = y\left(-d + \frac{bx}{y+x}\right)$$

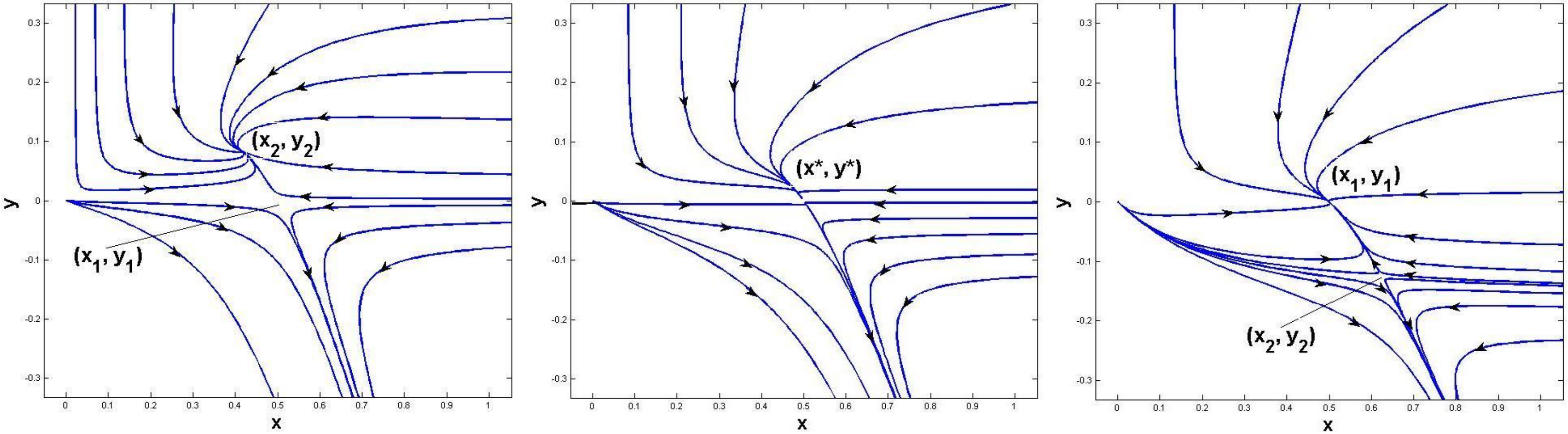
- The prey is harvested at a rate defined by a linear function.
- Two equilibria exist in the first quadrant under certain parameter values.
- One of the points represents coexistence of the species.
- Maximum Harvesting Effort = 1

Bifurcations in Model One

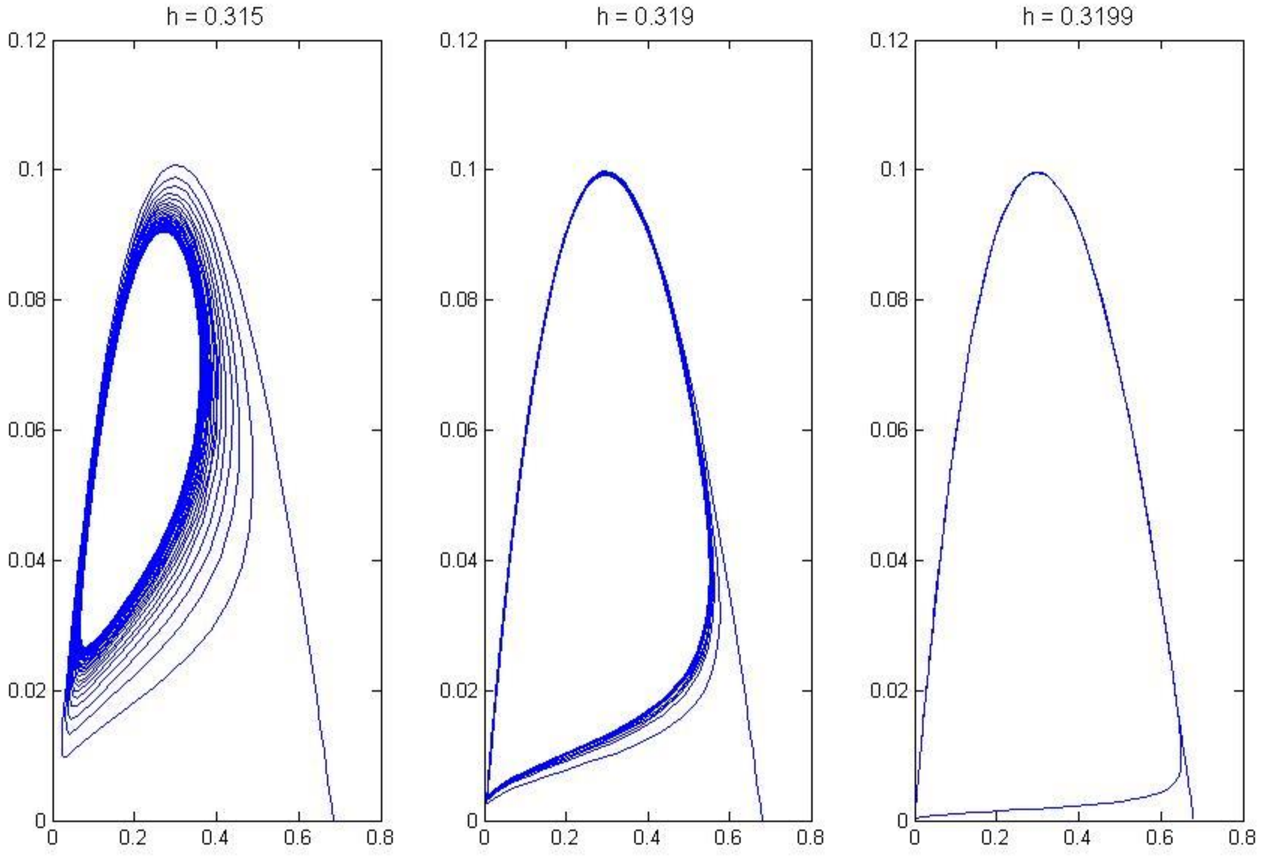
Hopf Bifurcation



Transcritical Bifurcation



Connecting Orbits



Model Two: Limit Harvesting

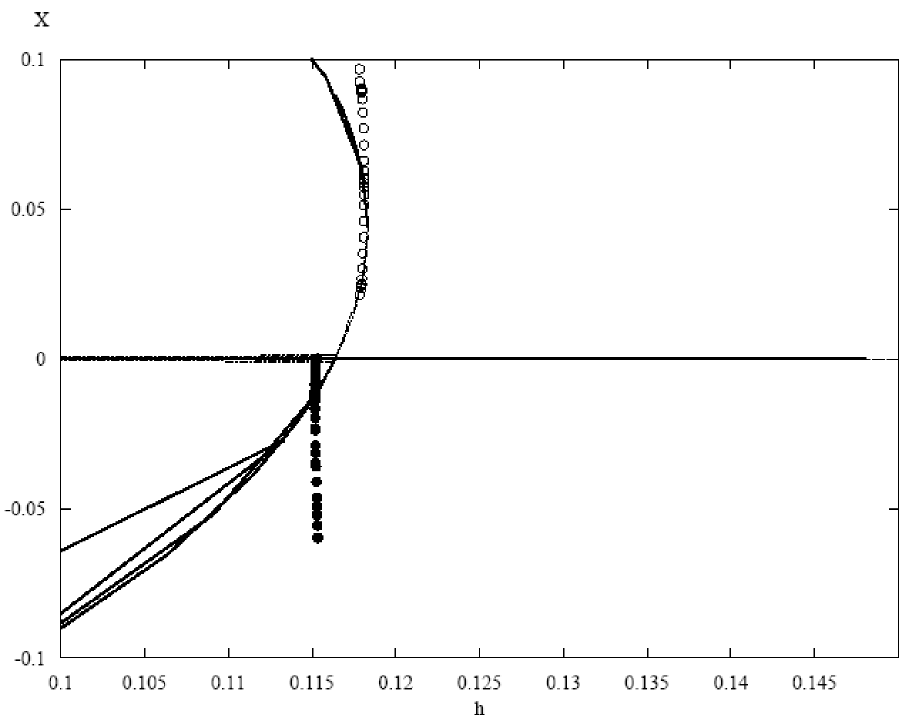
$$\dot{x} = x(1 - x) - \frac{axy}{y + x} - \frac{hx}{c + x}$$

$$\dot{y} = y\left(-d + \frac{bx}{y + x}\right)$$

- The prey is harvested at a rate defined by a rational function.
- The model has three equilibria that exist in the first quadrant under certain conditions.
- Again, one of the points represents coexistence of the species.

Bifurcations in Model Two

Bifurcation Diagram



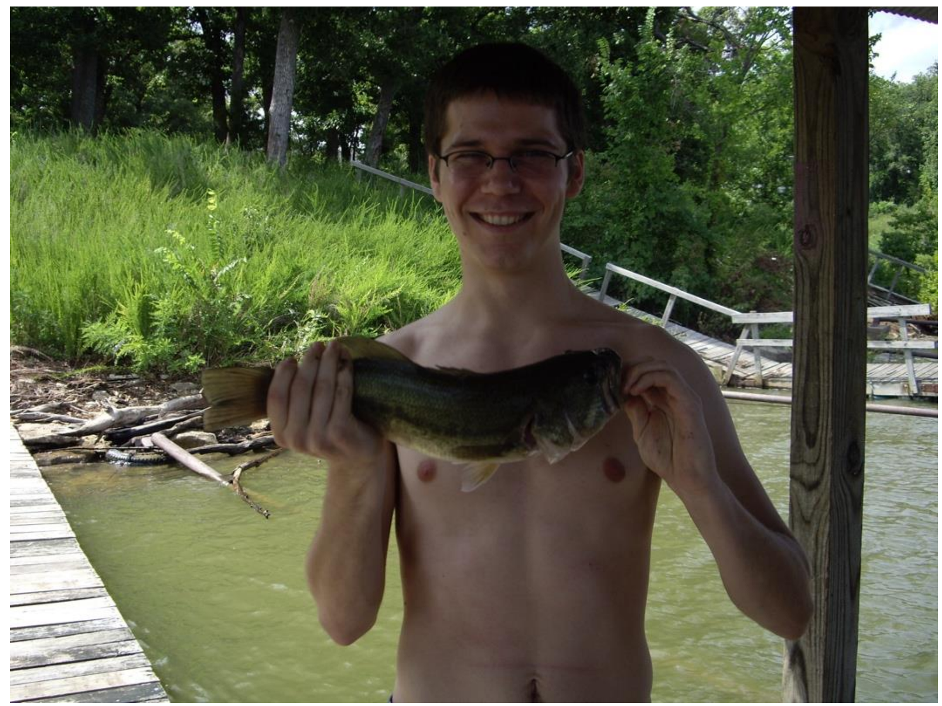
Conclusions

- Coexistence is possible under both harvesting policies.
- Multiple bifurcations and connecting orbits exist at the coexistence equilibria.
- Calculated Maximum Sustainable Yield for model one.

Future Research

- Study ratio-dependent models with other harvesting policies, such as seasonal harvesting.
- Investigate the dynamics of harvesting on the predator species or both species.
- Study the model with a harvesting agent who wishes to maximize its profit.

Real World Applications



Questions?