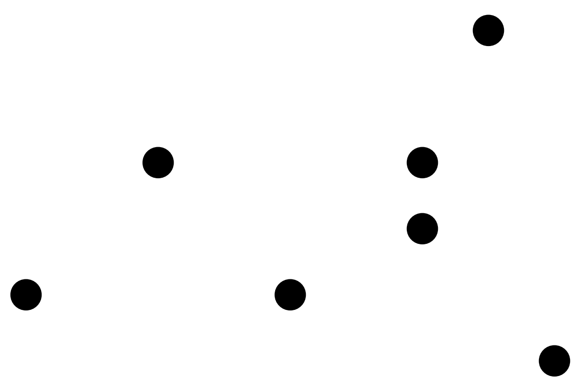
Spanning Trees in a Chair-Free Graph

Serenity Biggs
Warren Shull
University of Arkansas, Fayetteville

November 15, 2025

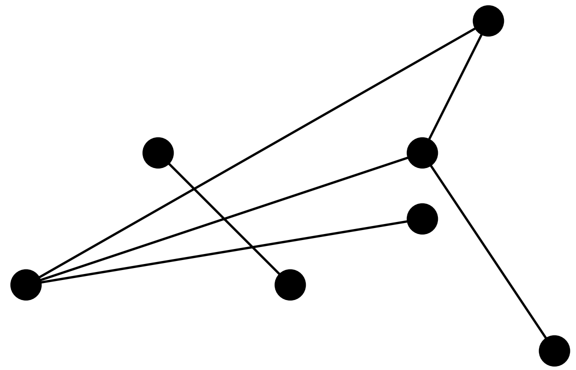
Basic Terminology

- Any set of objects, some of which have connections between them, can be thought of as a graph.
  - The word "graph" DOES NOT MEAN AN X OR Y AXIS in this context.
- The objects are shown as dots, or vertices (singular, vertex).



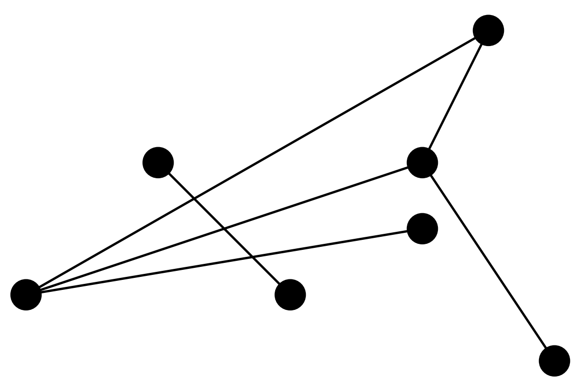
Basic Terminology

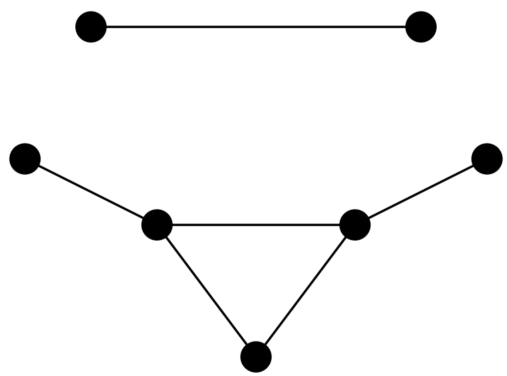
- Any set of objects, some of which have connections between them, can be thought of as a graph.
  - The word “graph” DOES NOT MEAN AN X OR Y AXIS in this context.
- The objects are shown as dots, or vertices (singular, vertex).
- The connections are shown as line segments, or edges.



Basic Terminology

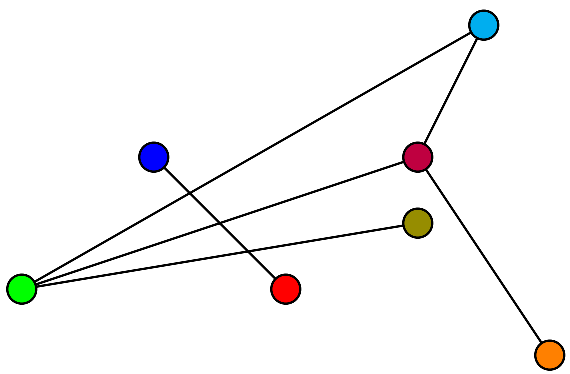
- Any set of objects, some of which have connections between them, can be thought of as a graph.
    - The word "graph" DOES NOT MEAN AN X OR Y AXIS in this context.
- The objects are shown as dots, or vertices (singular, vertex).
- The connections are shown as line segments, or edges.
- It doesn't matter where the vertices are drawn. It only matters which pairs of vertices are joined by edges.

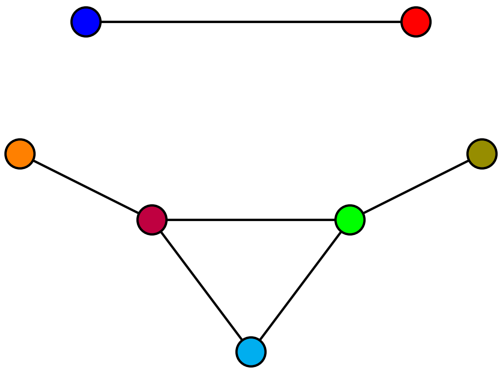




Basic Terminology

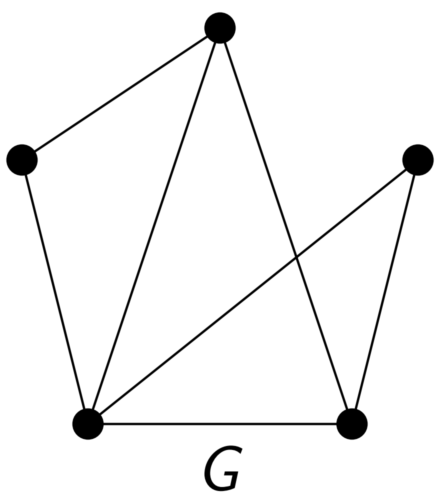
- Any set of objects, some of which have connections between them, can be thought of as a graph.
  - The word "graph" DOES NOT MEAN AN X OR Y AXIS in this context.
- The objects are shown as dots, or vertices (singular, vertex).
- The connections are shown as line segments, or edges.
- It doesn't matter where the vertices are drawn. It only matters which pairs of vertices are joined by edges.

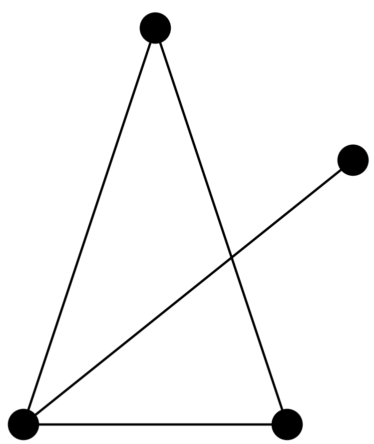




Basic Terminology

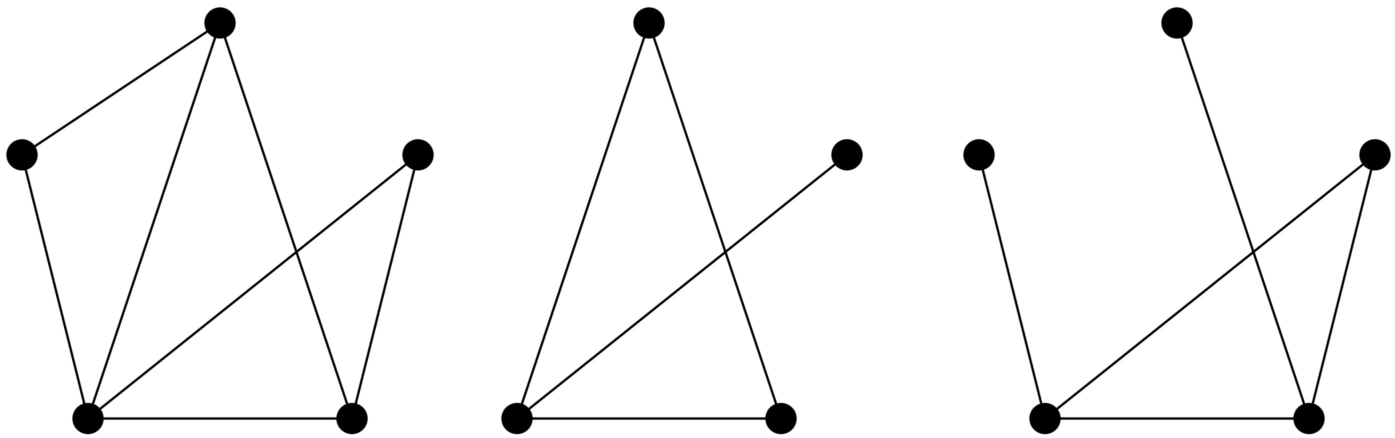
- A subgraph of a graph G is another graph that contains some, but possibly not all, of the vertices and edges of G.





Basic Terminology

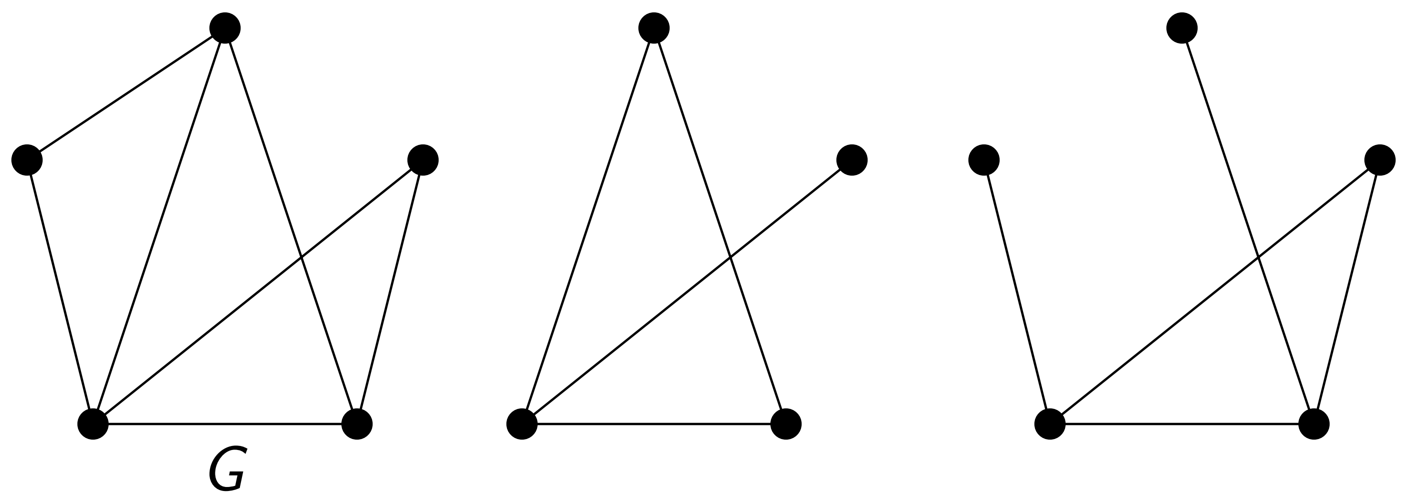
- A subgraph of a graph G is another graph that contains some, but possibly not all, of the vertices and edges of G.
- If it contains all the vertices, we call it a spanning subgraph.



G

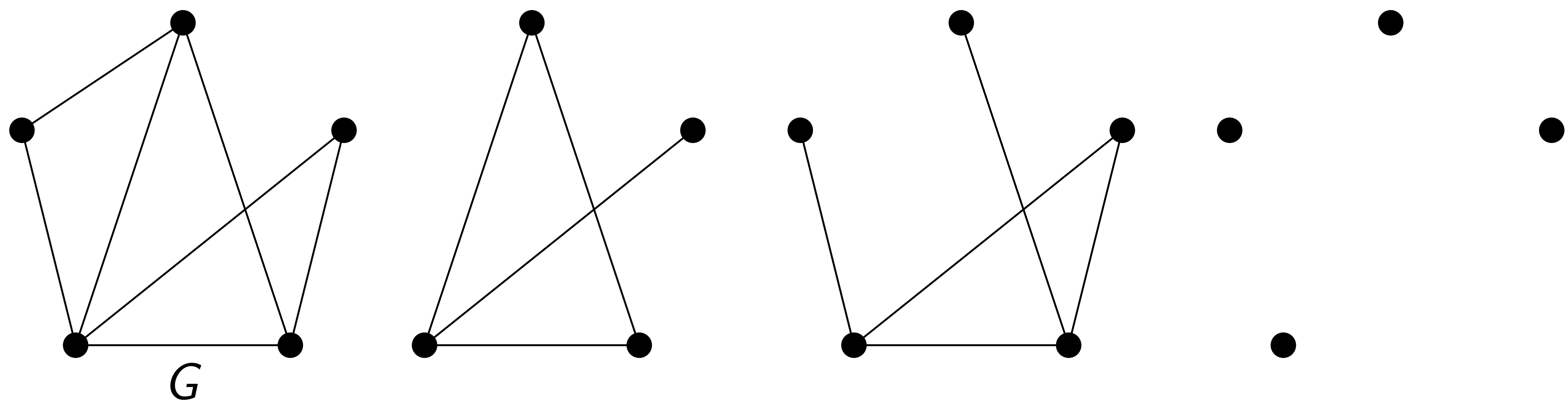
Basic Terminology

- A <u>subgraph</u> of a graph G is another graph that contains some, but possibly not all, of the vertices and edges of G.
- If it contains all the vertices, we call it a <u>spanning subgraph</u>.
- A subgraph is called <u>induced</u> if it contains every edge whose endpoints are both in its vertex set.



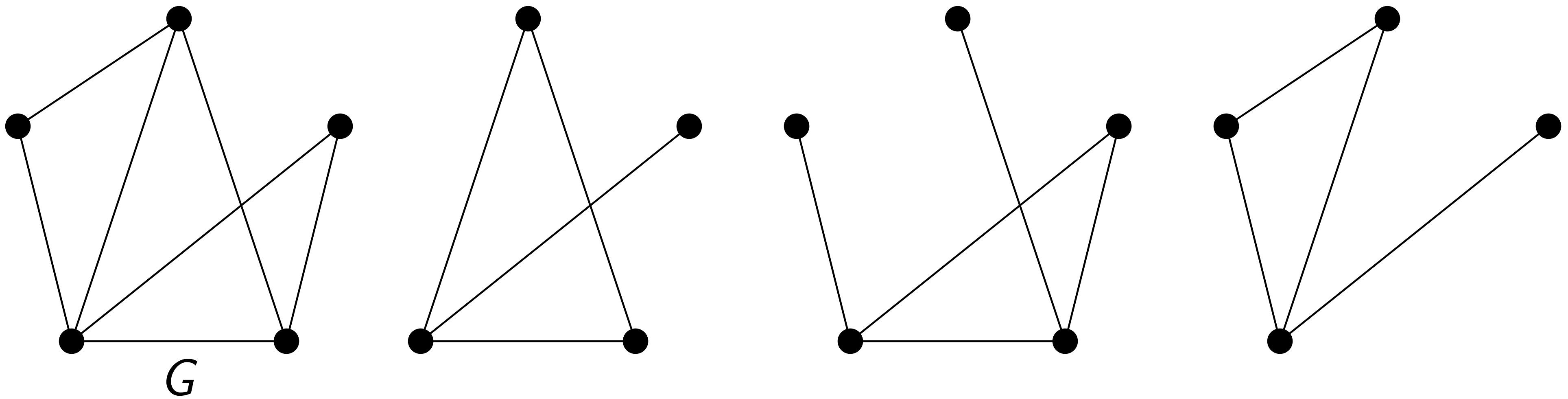
Basic Terminology

- A <u>subgraph</u> of a graph G is another graph that contains some, but possibly not all, of the vertices and edges of G.
- If it contains all the vertices, we call it a <u>spanning subgraph</u>.
- A subgraph is called <u>induced</u> if it contains every edge whose endpoints are both in its vertex set.



Basic Terminology

- A subgraph of a graph G is another graph that contains some, but possibly not all, of the vertices and edges of G.
- If it contains all the vertices, we call it a spanning subgraph.
- A subgraph is called induced if it contains every edge whose endpoints are both in its vertex set.



Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.

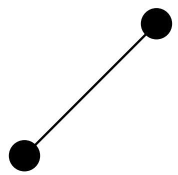
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.

•

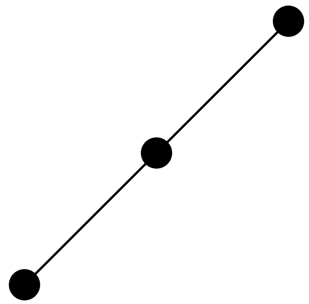
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.



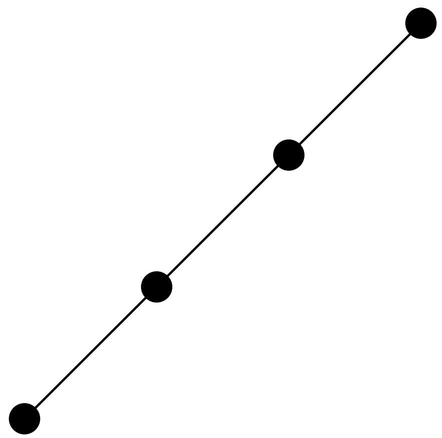
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.



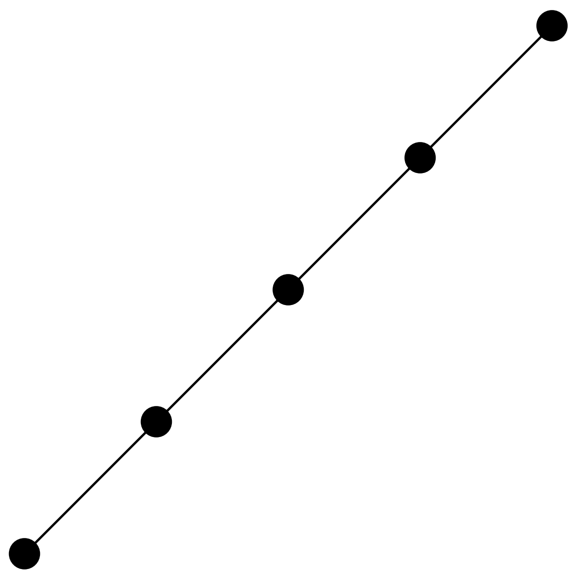
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.



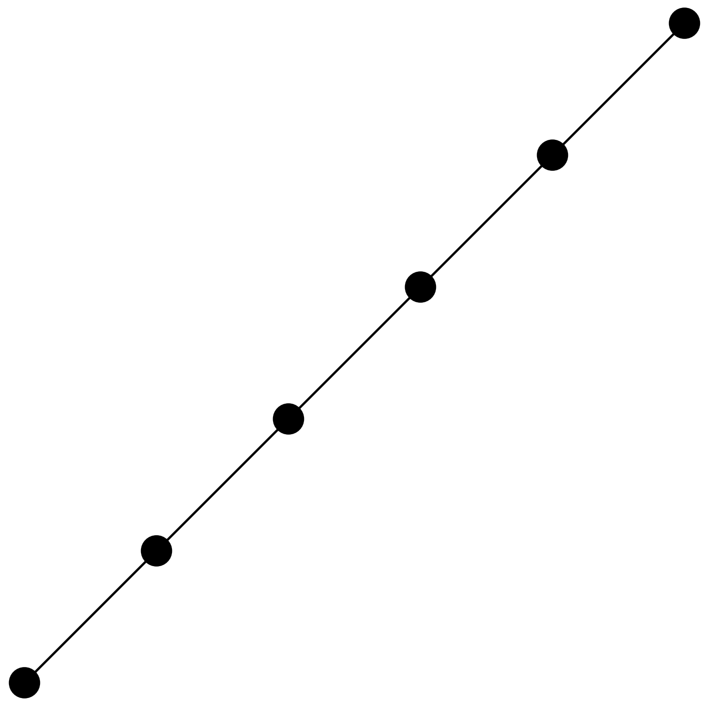
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.



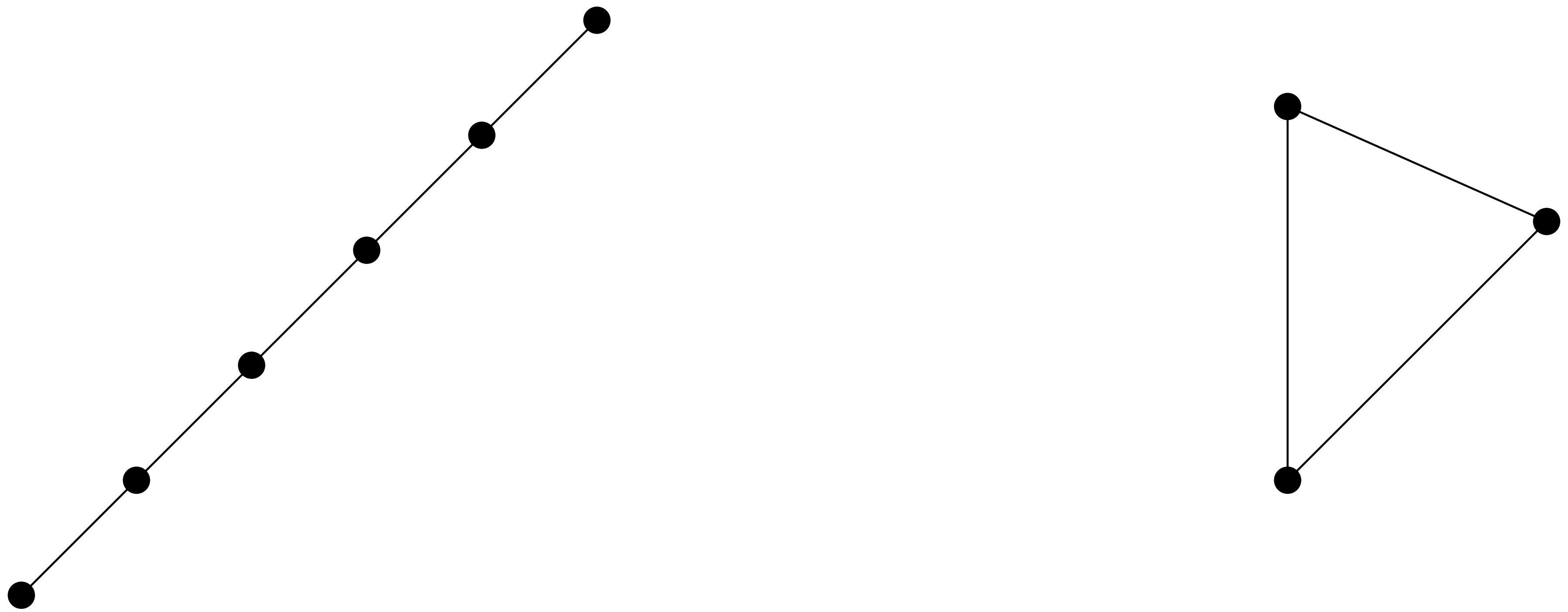
Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.



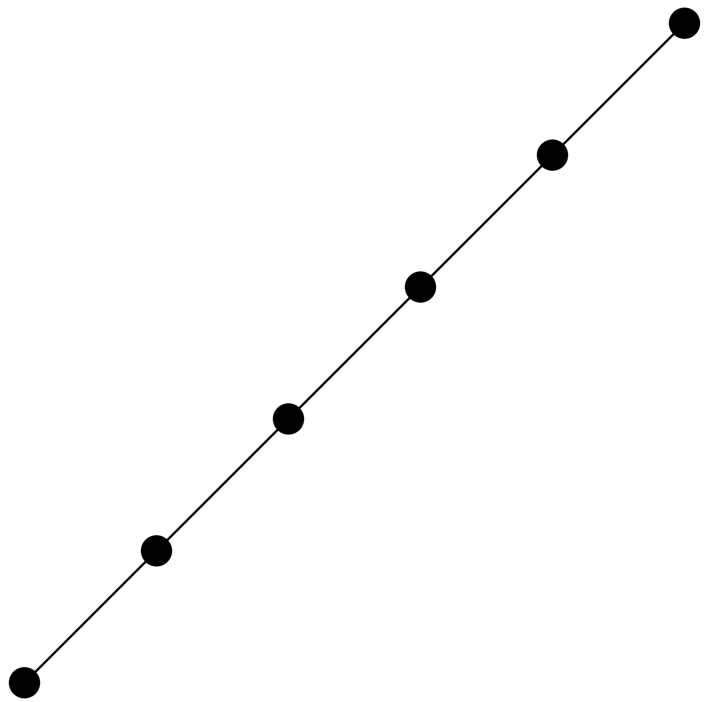
Basic Terminology

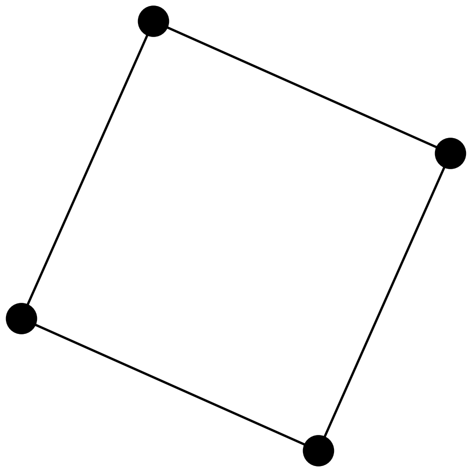
- The words path, cycle, and connected mean essentially what they sound like.



Basic Terminology

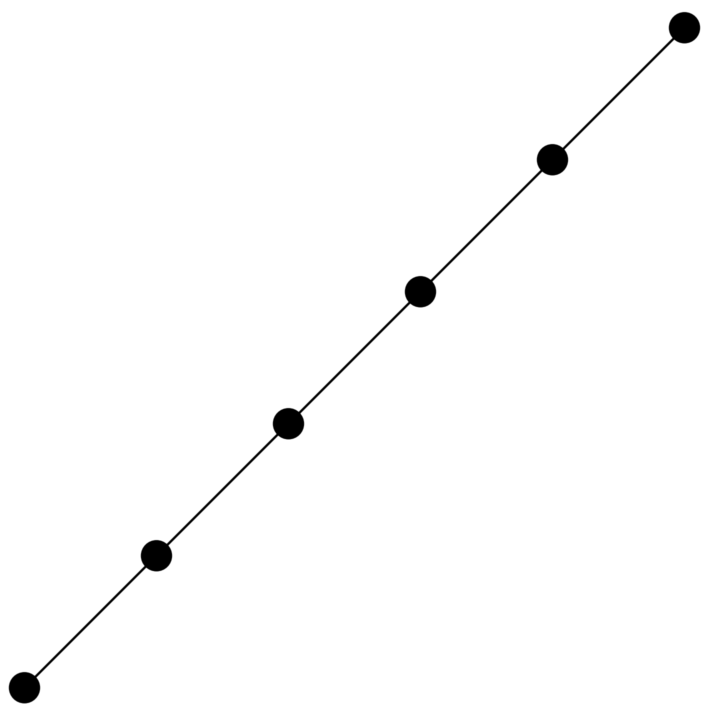
- The words path, cycle, and connected mean essentially what they sound like.

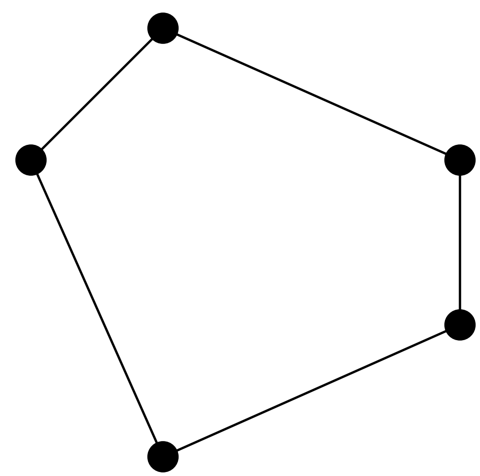




Basic Terminology

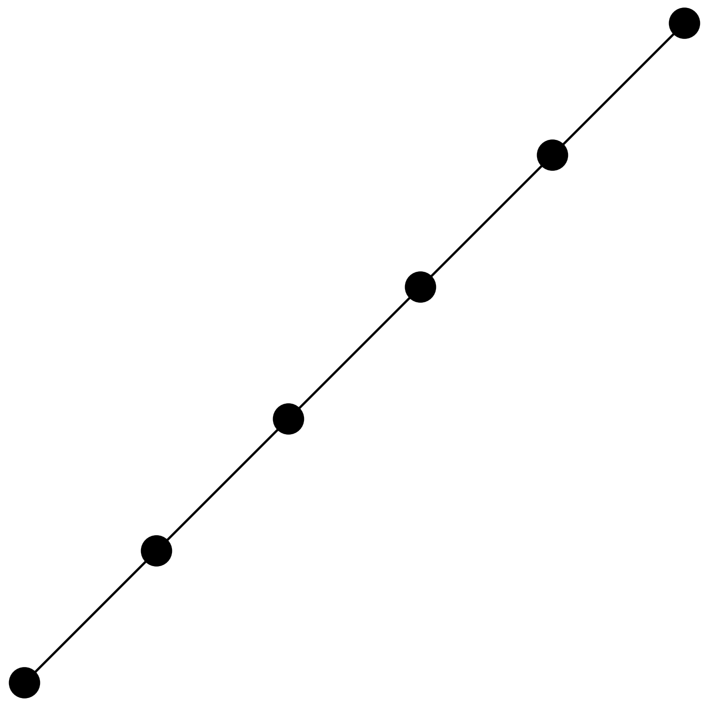
- The words path, cycle, and connected mean essentially what they sound like.

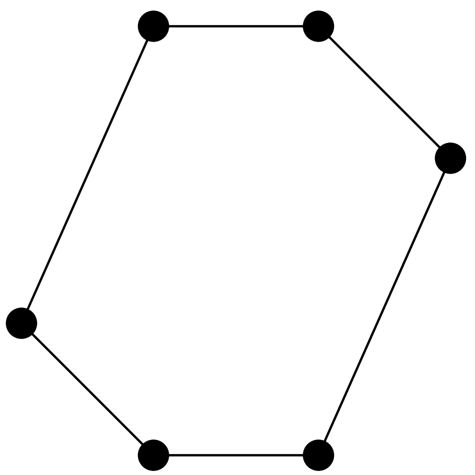




Basic Terminology

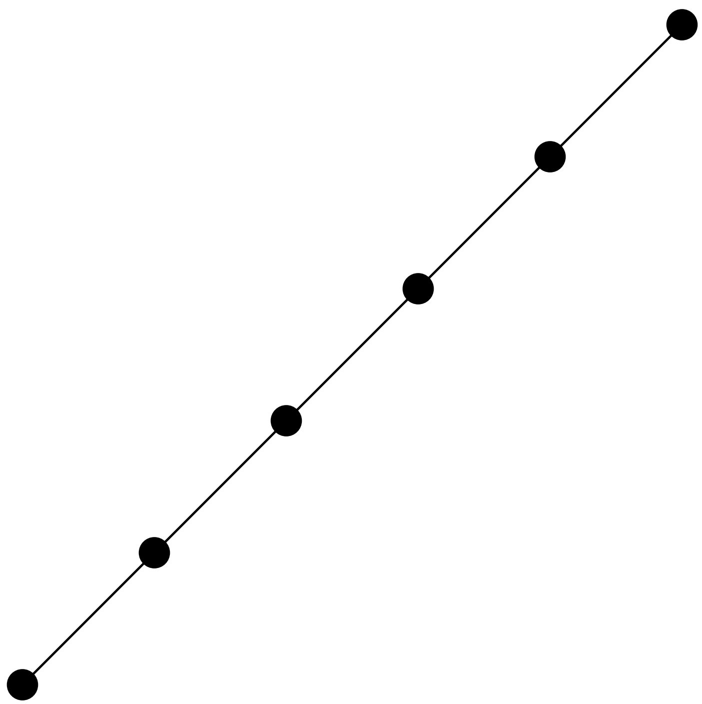
- The words path, cycle, and connected mean essentially what they sound like.

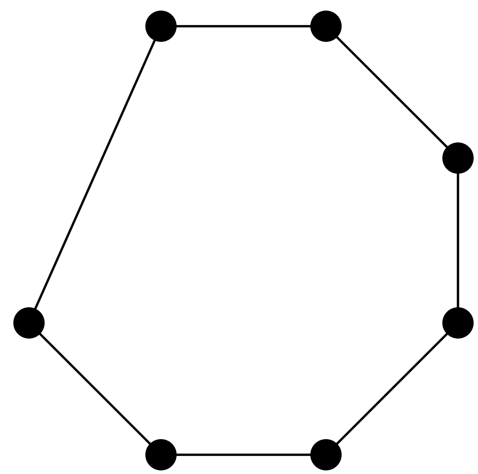




Basic Terminology

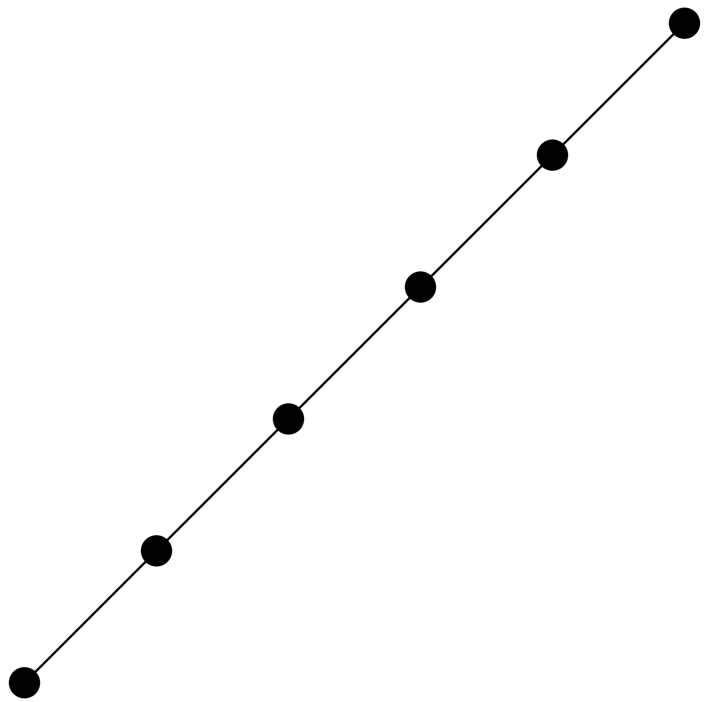
- The words path, cycle, and connected mean essentially what they sound like.

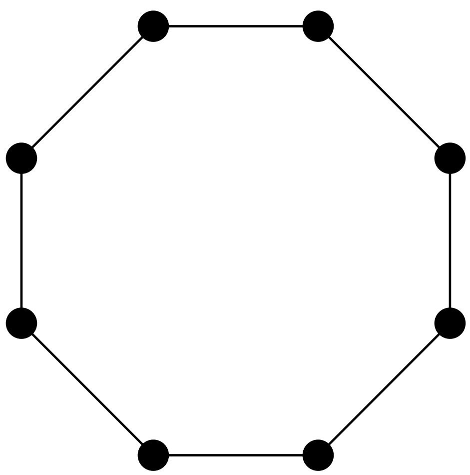




Basic Terminology

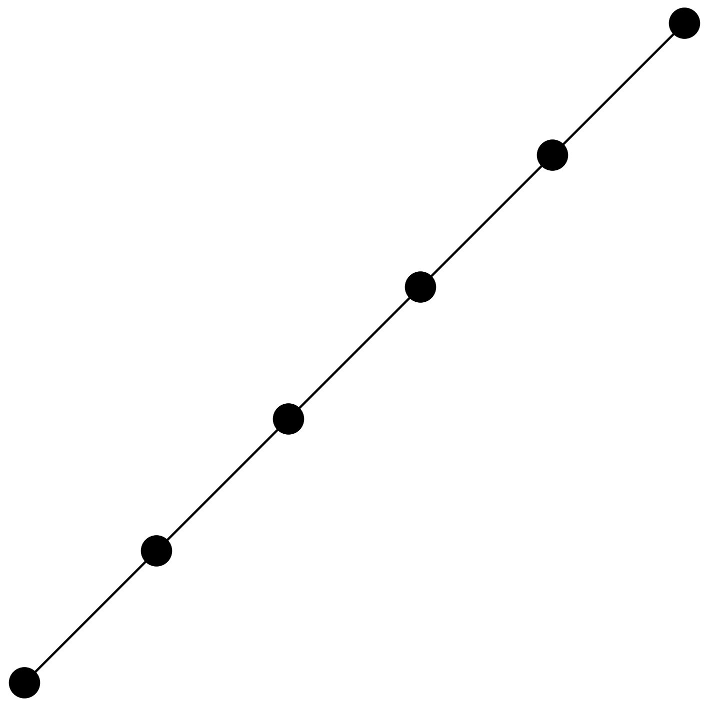
- The words path, cycle, and connected mean essentially what they sound like.

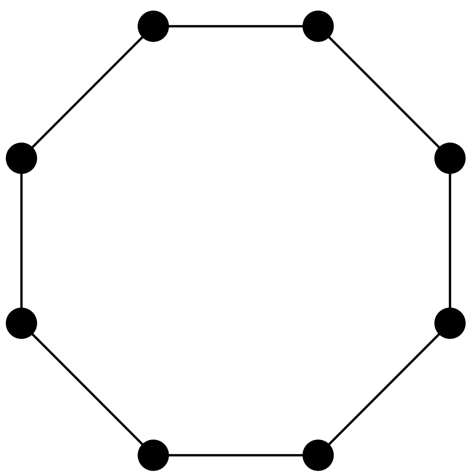




Basic Terminology

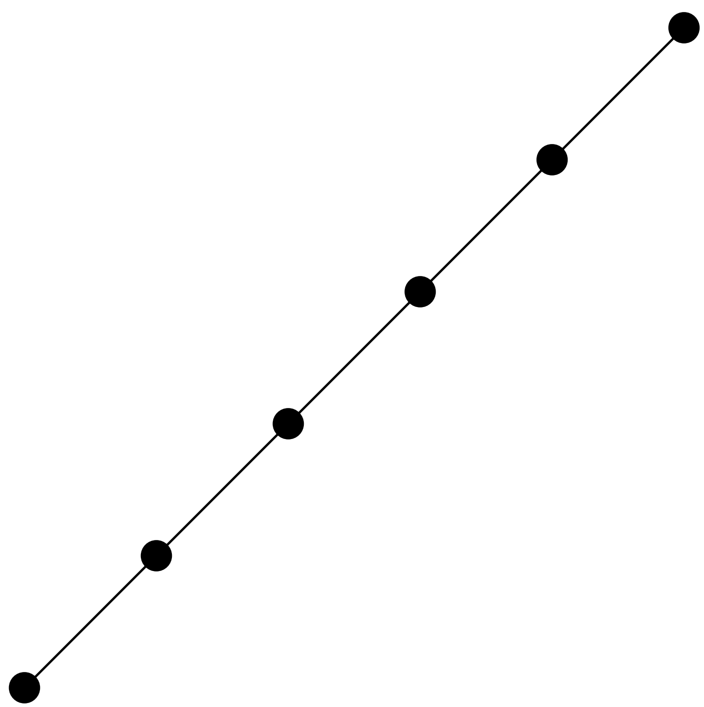
- The words path, cycle, and connected mean essentially what they sound like.
- A tree is a connected graph with no cycles.
  - Note that paths are a special kind of tree.

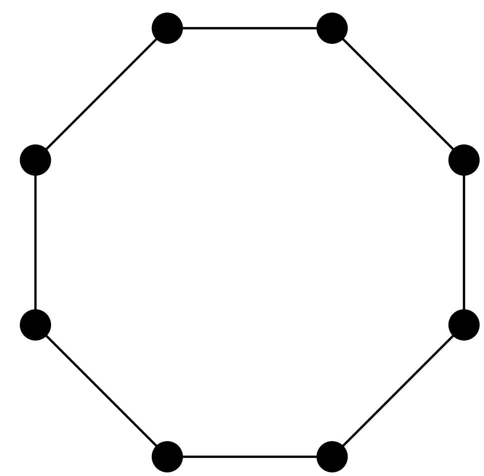




Basic Terminology

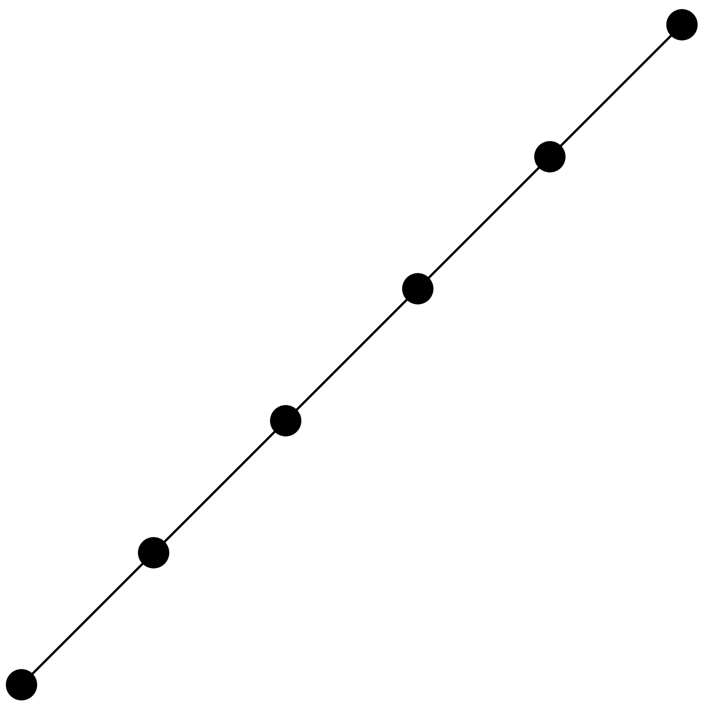
- The words path, cycle, and connected mean essentially what they sound like.
- A tree is a connected graph with no cycles.
  - Note that paths are a special kind of tree.
- If a tree is a spanning subgraph, we call it a spanning tree.
  - Every connected graph has one.

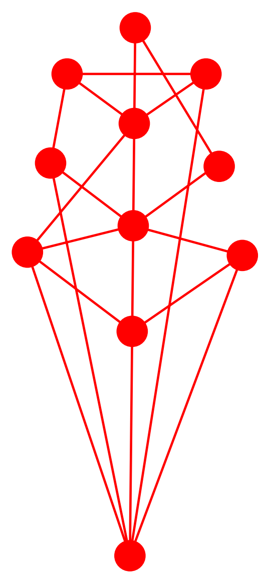


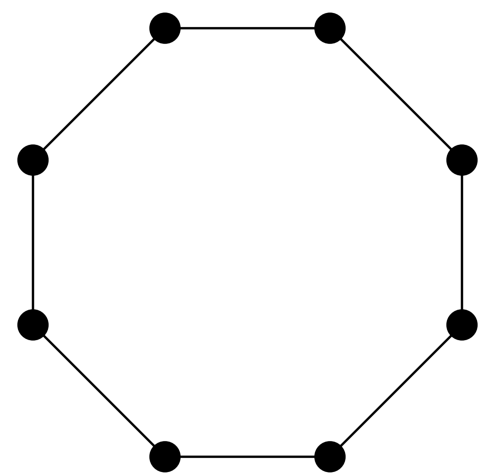


Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.
- A tree is a connected graph with no cycles.
  - Note that paths are a special kind of tree.
- If a tree is a spanning subgraph, we call it a spanning tree.
  - Every connected graph has one.

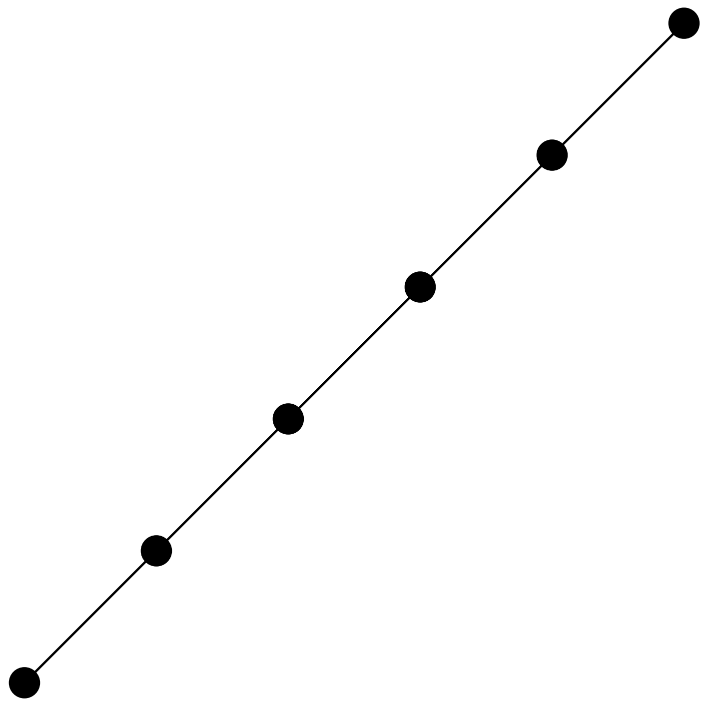


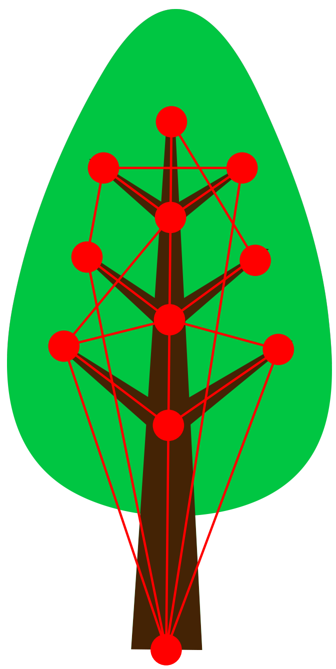


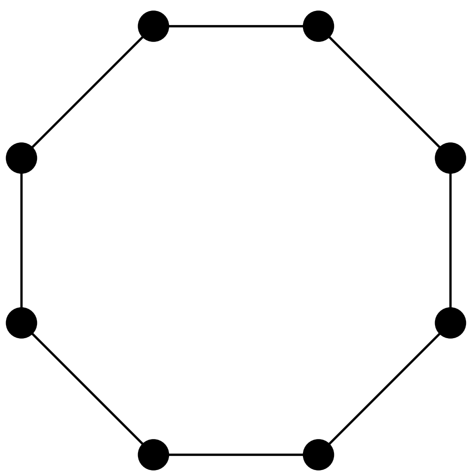


Basic Terminology

- The words path, cycle, and connected mean essentially what they sound like.
- A tree is a connected graph with no cycles.
  - Note that paths are a special kind of tree.
- If a tree is a spanning subgraph, we call it a spanning tree.
  - Every connected graph has one.

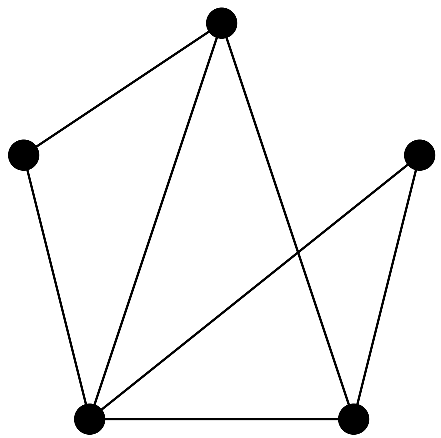






Spanning trees

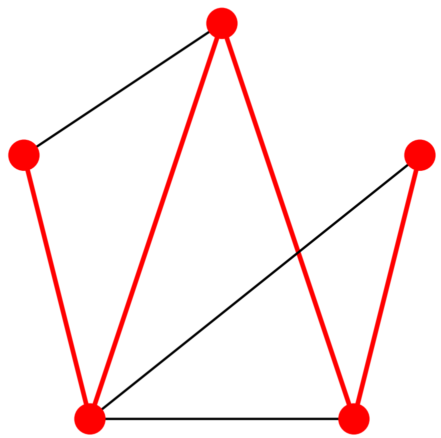
- Recall that paths are a special kind of spanning tree.
- If a spanning tree is a path, we call it a Hamiltonian path.



G

Spanning trees

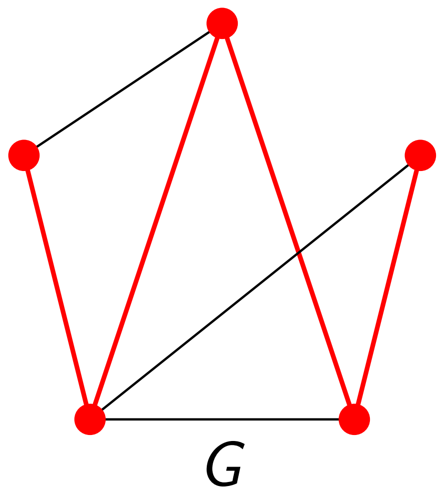
- Recall that paths are a special kind of spanning tree.
- If a spanning tree is a path, we call it a Hamiltonian path.

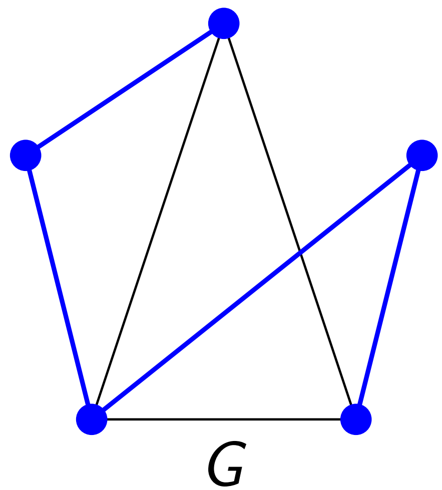


G

Spanning trees

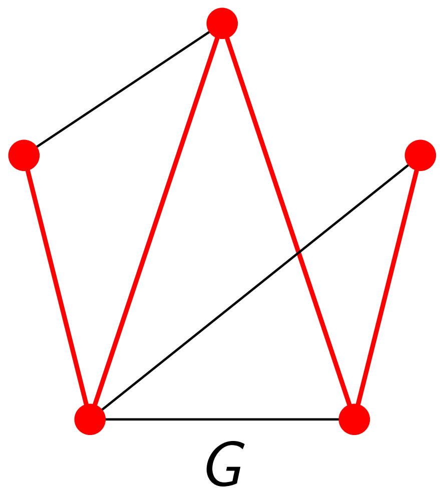
- Recall that paths are a special kind of spanning tree.
- If a spanning tree is a path, we call it a Hamiltonian path.

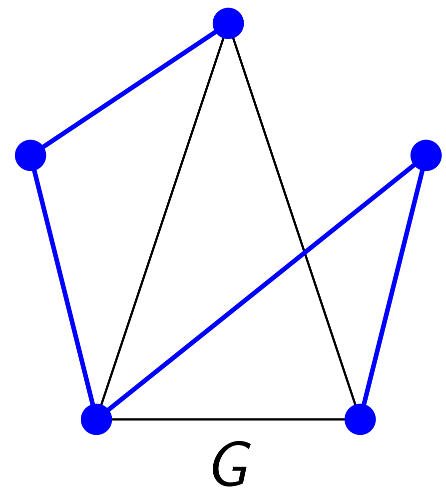




Spanning trees

- Recall that paths are a special kind of spanning tree.
- If a spanning tree is a path, we call it a Hamiltonian path.
- As graphs get larger, checking for a Hamiltonian path takes way too long, even with a computer.
- Like many researchers, we focus on sufficient conditions.





Spanning trees

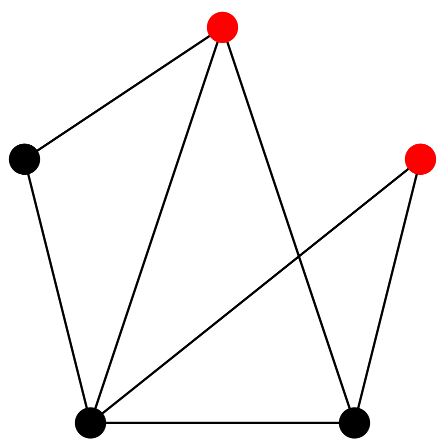
- The degree of a vertex is the number of edges coming out of it.
- Leaf of a tree: degree 1
- Branch vertex of a tree: degree 3
- Why are paths so special?
  - Max degree 2
  - 2 leaves
  - No branch vertices
- Some spanning trees are "close" to being a Hamiltonian path, in a few different ways:
  - Low maximum degree
  - Few leaves
  - **Few branch vertices**

Throughout this talk, we prefer spanning trees with fewer branch vertices.

- What conditions might lead to better spanning trees?

Independent sets

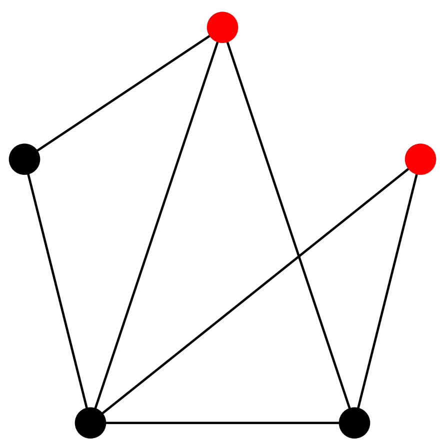
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?



G

Independent sets

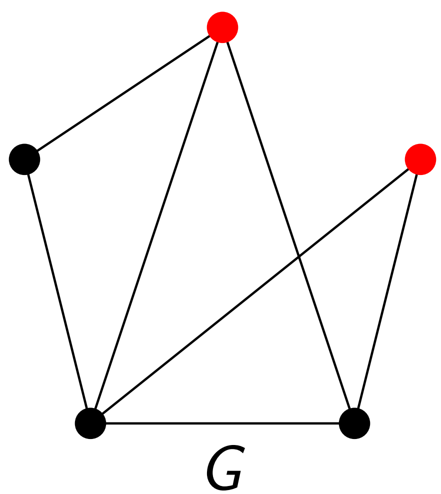
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.



G

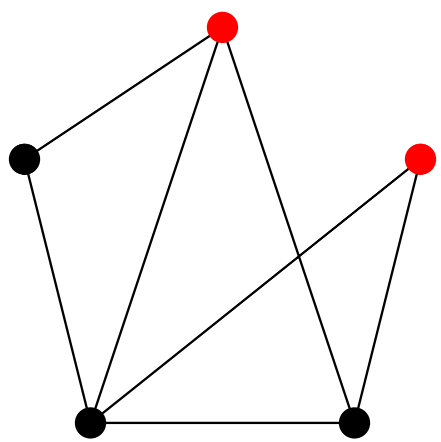
Independent sets

- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.
    - $3 + 2 = 5$



Independent sets

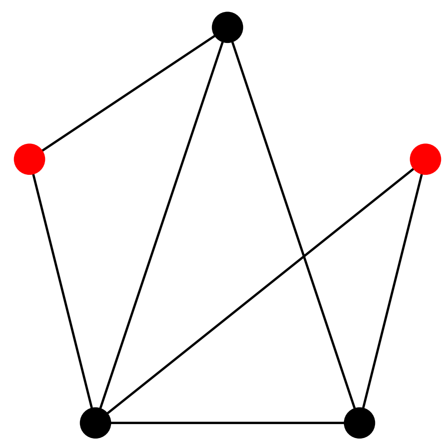
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.
  - $3 + 2 = 5$
- Can you find an equally large independent set with smaller total degree?



G

Independent sets

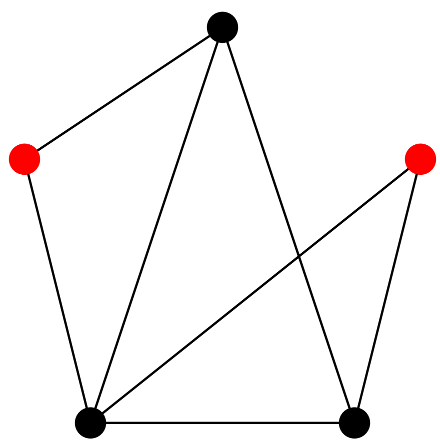
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.
  - $3 + 2 = 5$
- Can you find an equally large independent set with smaller total degree?



G

Independent sets

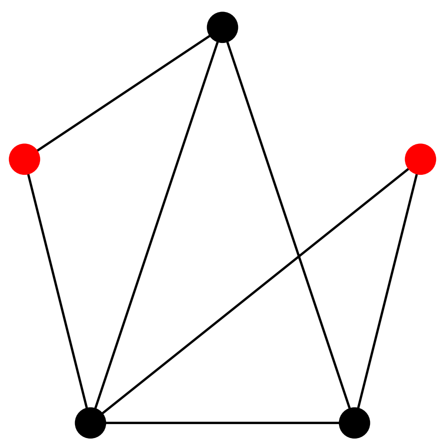
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.
  - $3 + 2 = 5$
- Can you find an equally large independent set with smaller total degree?
  - $2 + 2 = 4$



G

Independent sets

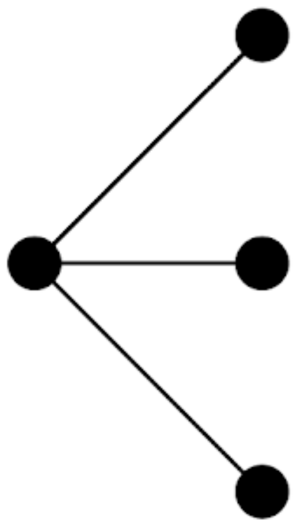
- In a graph G, a collection of vertices with no edges between them is called an independent set.
- Do you see any larger independent sets in this graph?
- Add their degrees together. This is called **total degree**.
  - $3 + 2 = 5$
- Can you find an equally large independent set with smaller total degree?
  - $2 + 2 = 4$
- Keep this goal in mind: find the largest independent set, and make its total degree as small as possible.

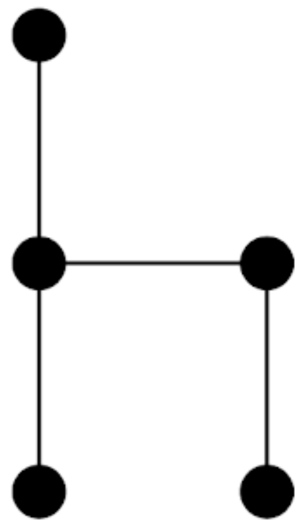


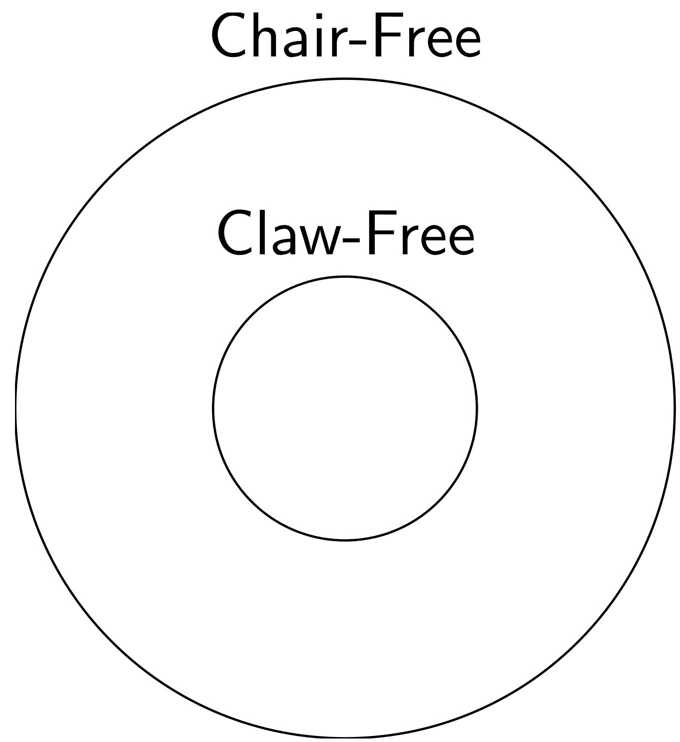
G

Claws and Chairs

- A graph with no induced claw (chair) is called claw-free (chair-free).
- Spanning trees have been extensively studied in claw-free graphs, but very little in chair-free graphs.
- "Most" chair-free graphs seem to also be claw-free.







Independent sets v. spanning trees

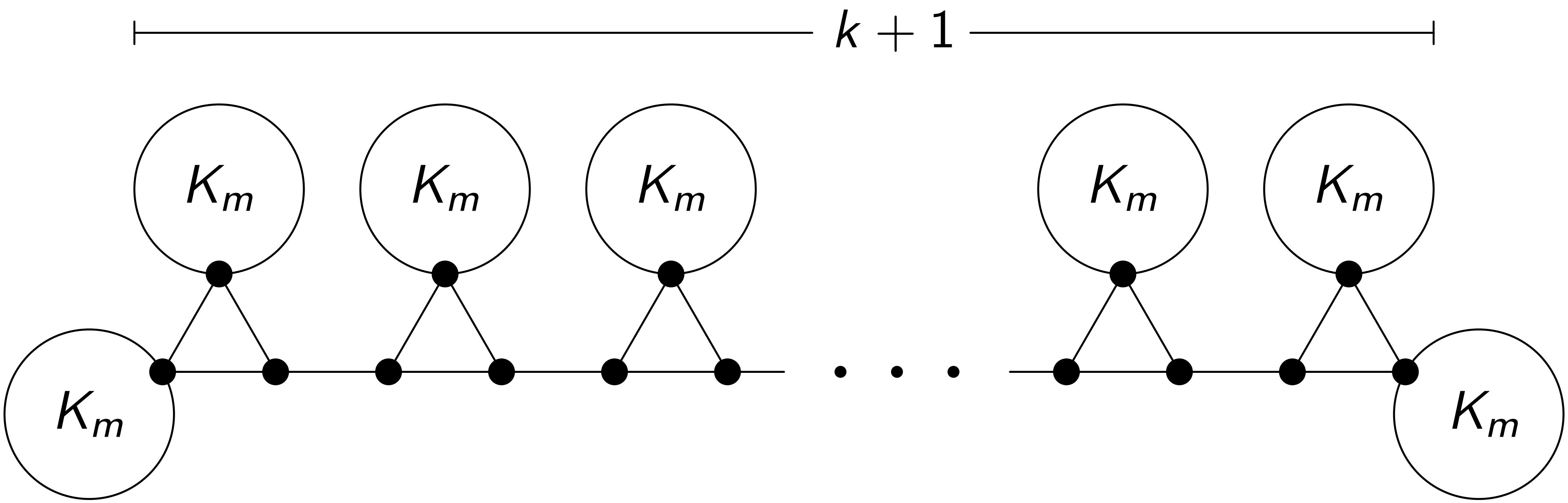
- adding edges creates more options for better spanning trees
- deleting edges makes larger independent sets with smaller total degree

Goal: Guarantee a spanning tree with at most ______ branch vertices or a ______-vertex independent set with total degree at most ______

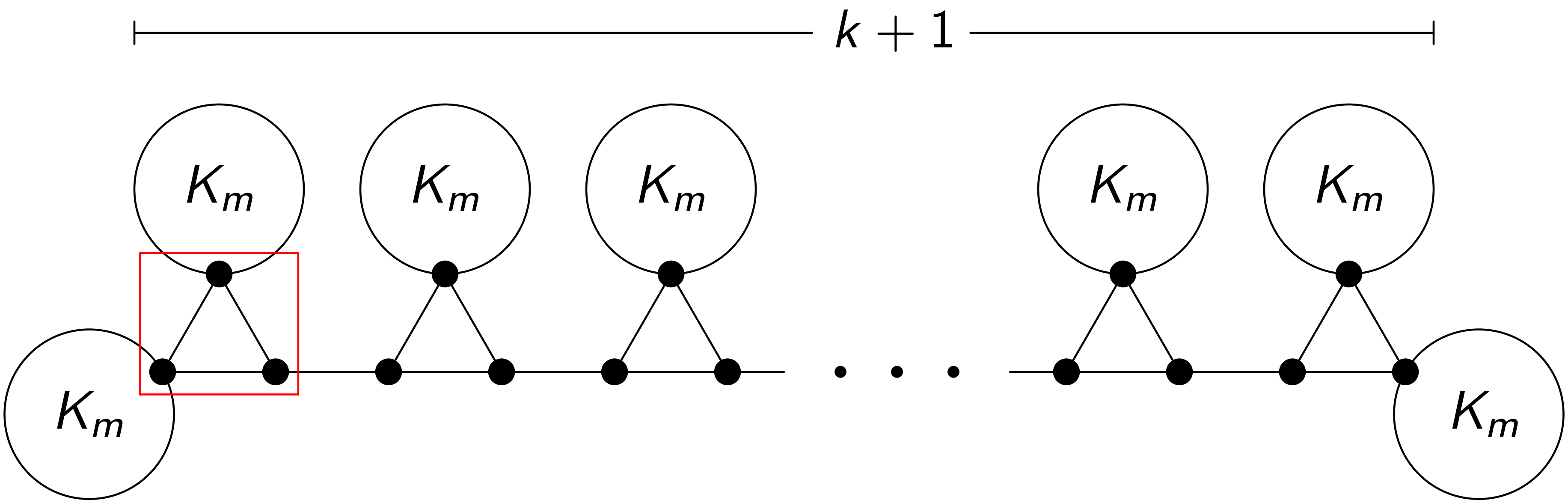
We want to fill in the blanks so that, if an enemy removes enough edges to take away the tree, it must create the independent set (and vice versa).

If the red numbers were large or the green number was small, this would be easy. But we want **small red numbers** and a **large green number**.

If the graph is claw-free or chair-free, what are the best possible numbers?

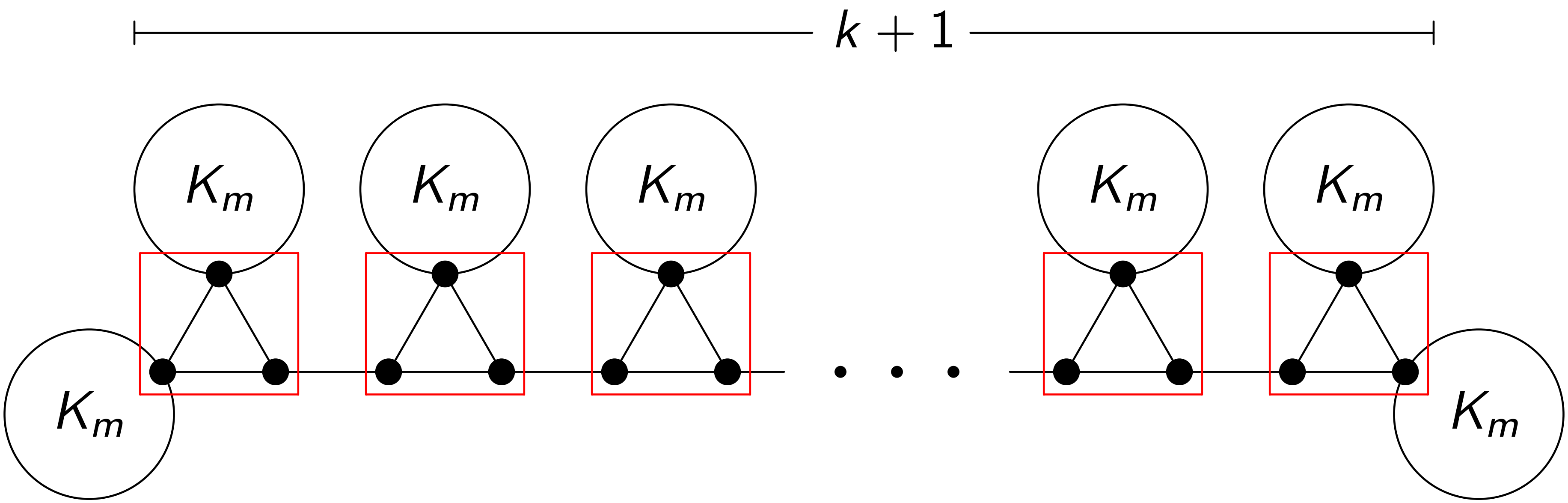


Connected, claw-free, and chair-free



Connected, claw-free, and chair-free

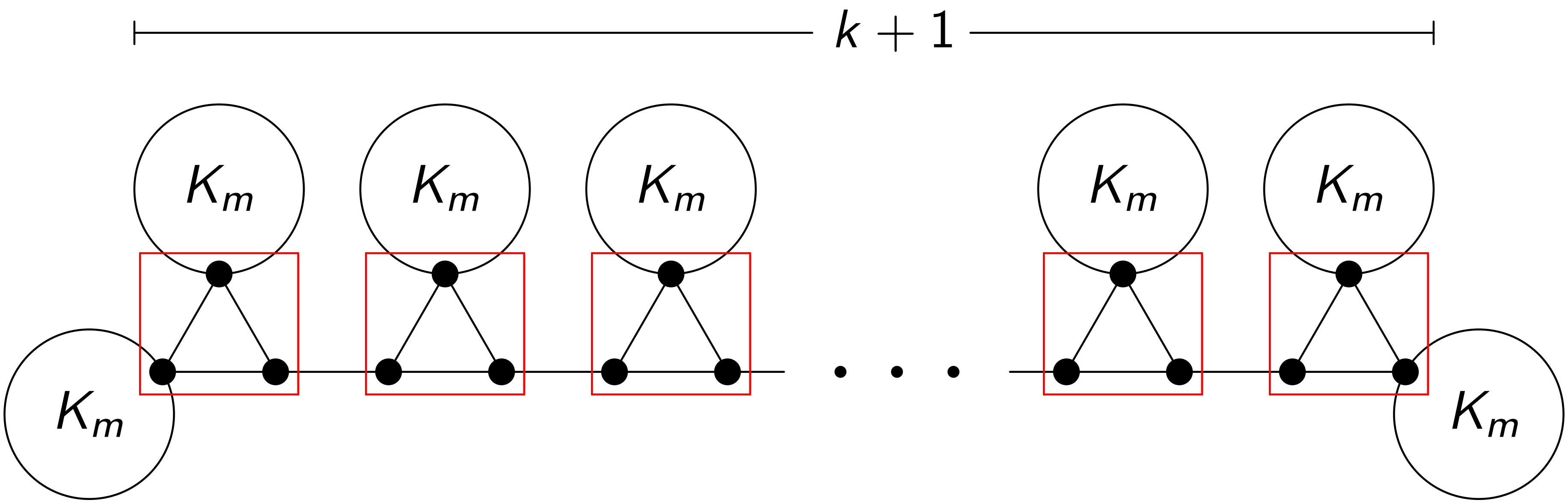
Any spanning tree must have a branch vertex in this triangle...



Connected, claw-free, and chair-free

Any spanning tree must have a branch vertex in this triangle...

...and each of these others...

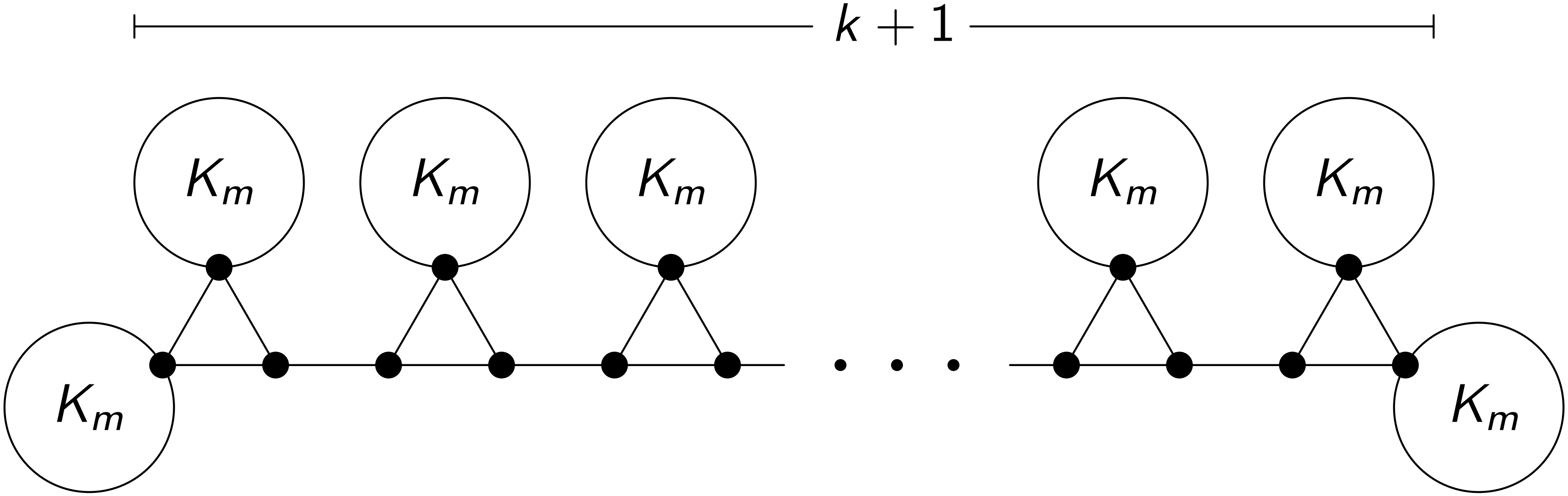


Connected, claw-free, and chair-free

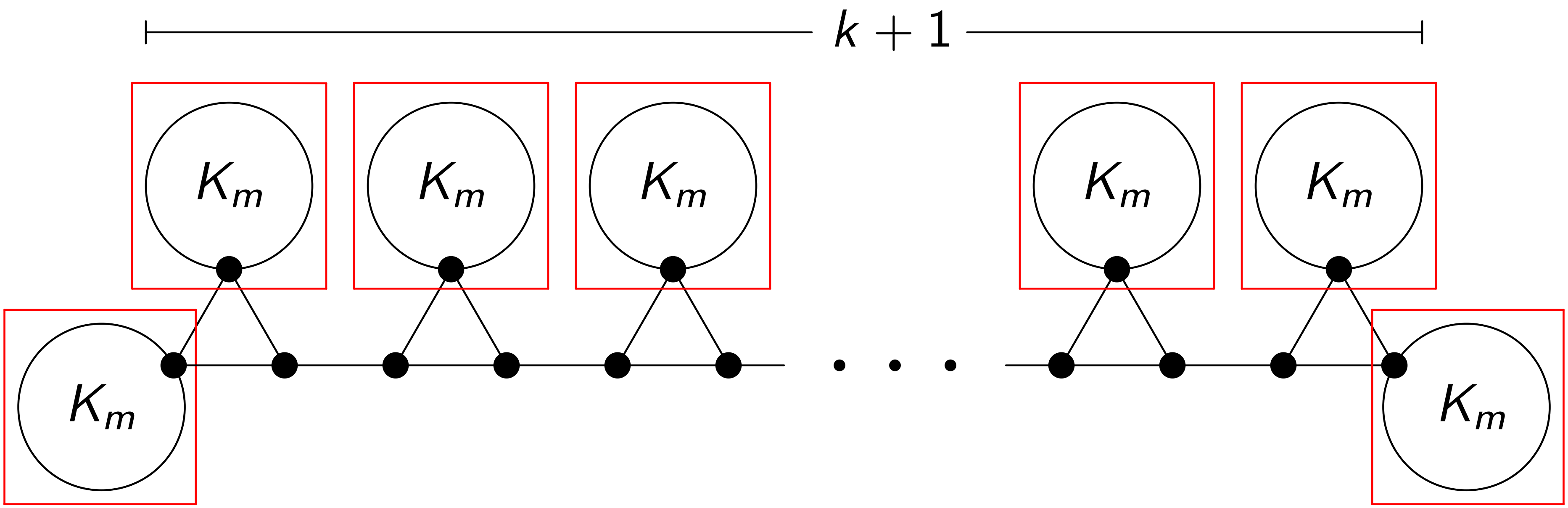
Any spanning tree must have a branch vertex in this triangle...

...and each of these others...

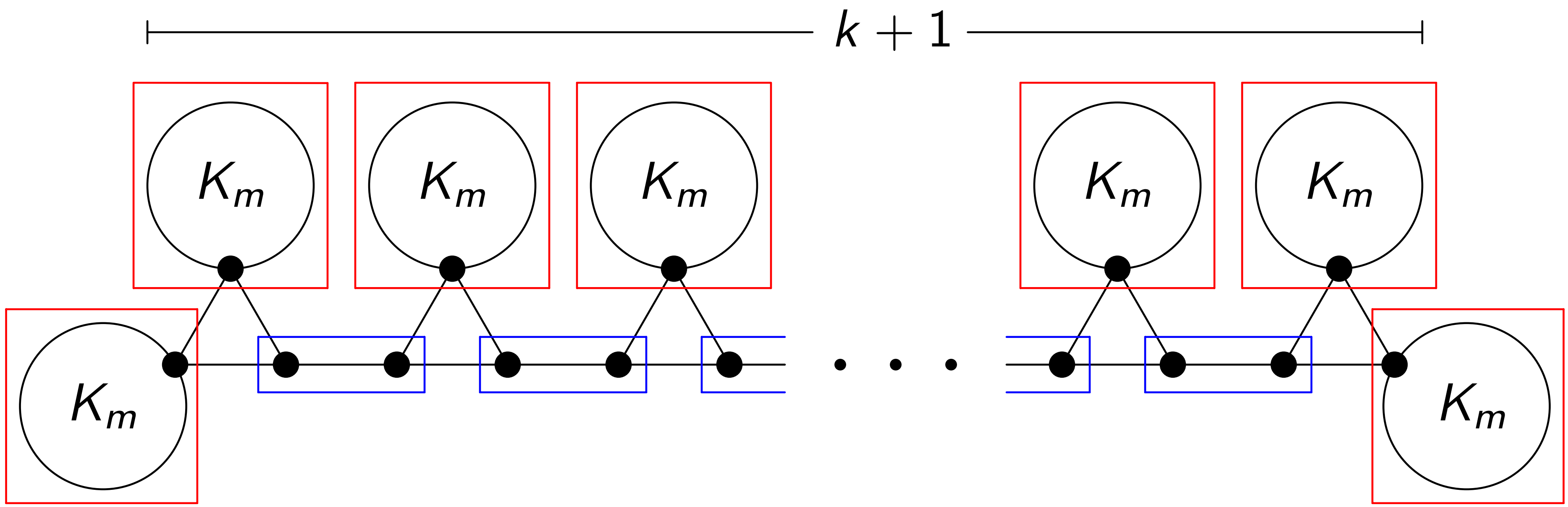
...for a minimum of $k+1$ branch vertices.



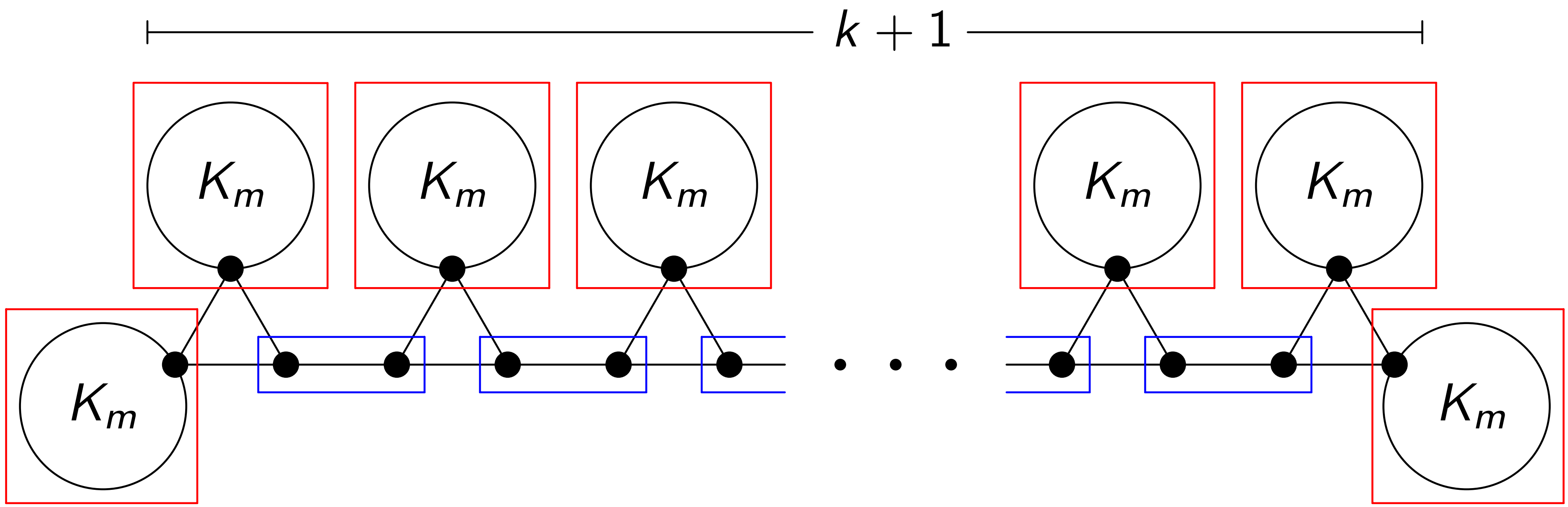
$$|V(G)| \quad =$$



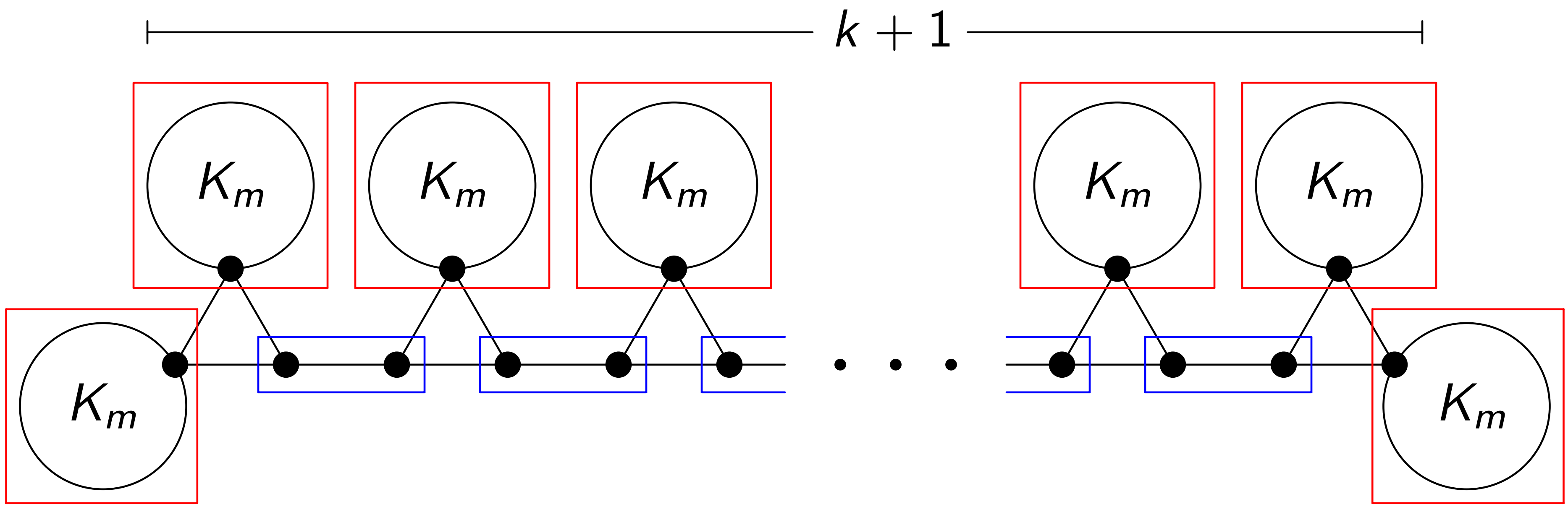
$$|V(G)| \quad = \quad m(k+3)$$



$$|V(G)| = m(k+3) + 2k$$

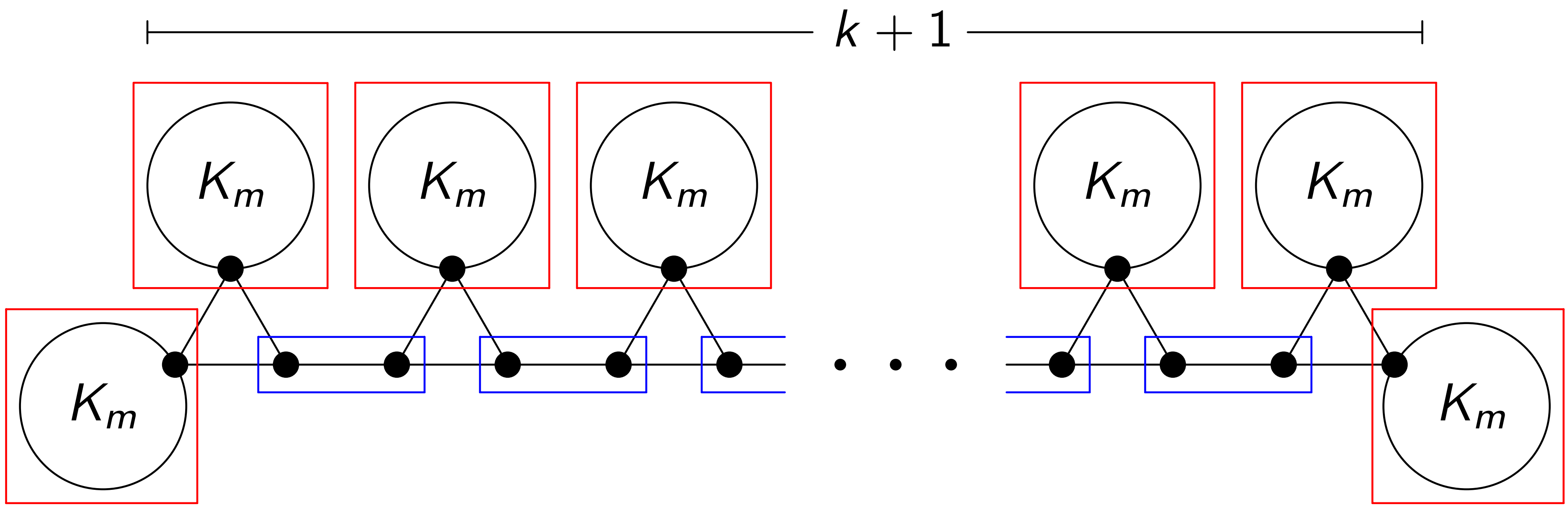


$$|V(G)| \quad = \quad m(k+3) + 2k = mk + 3m + 2k$$



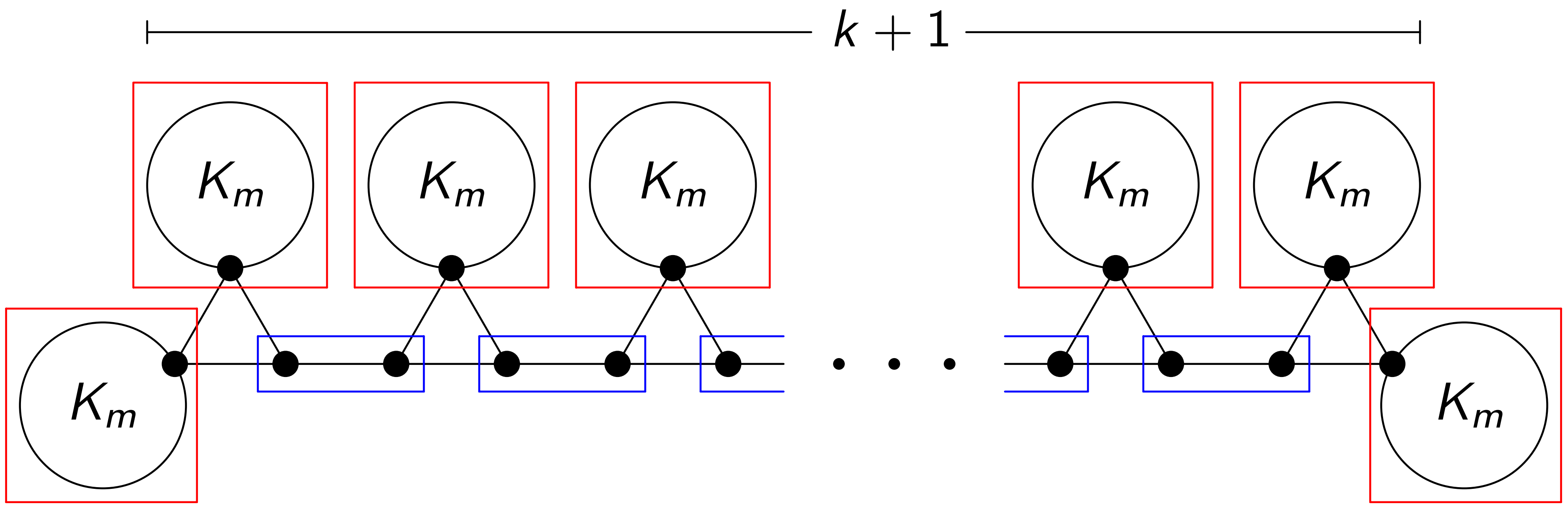
$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

$$|X| \leq$$

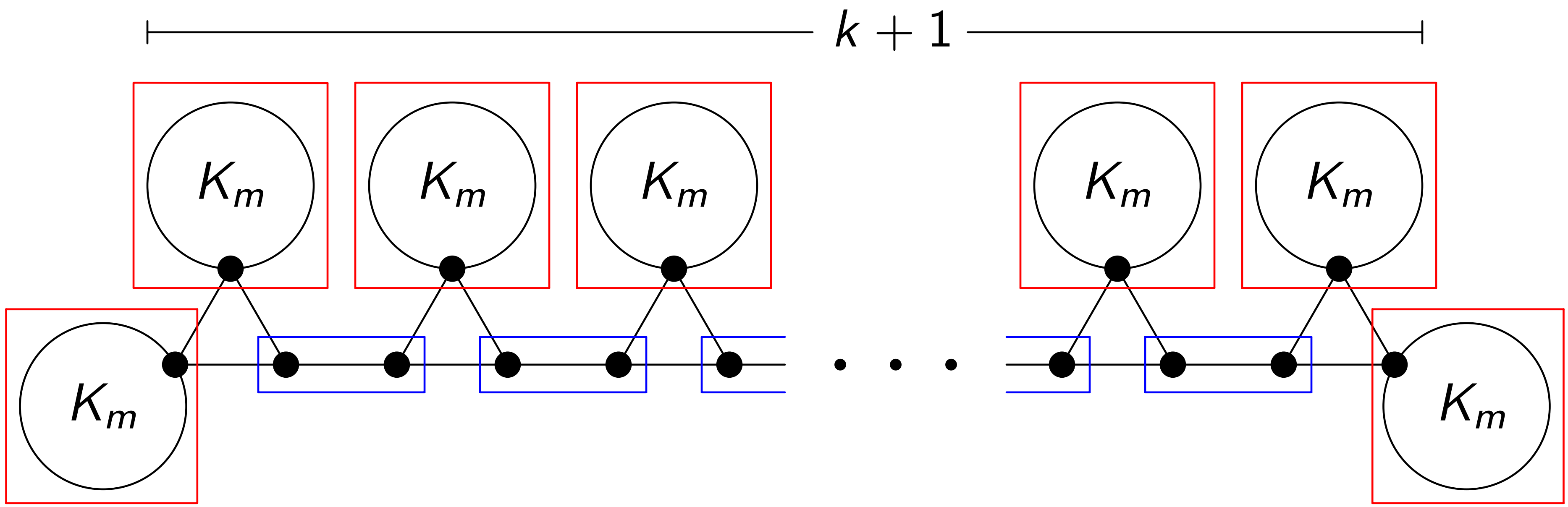


$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$

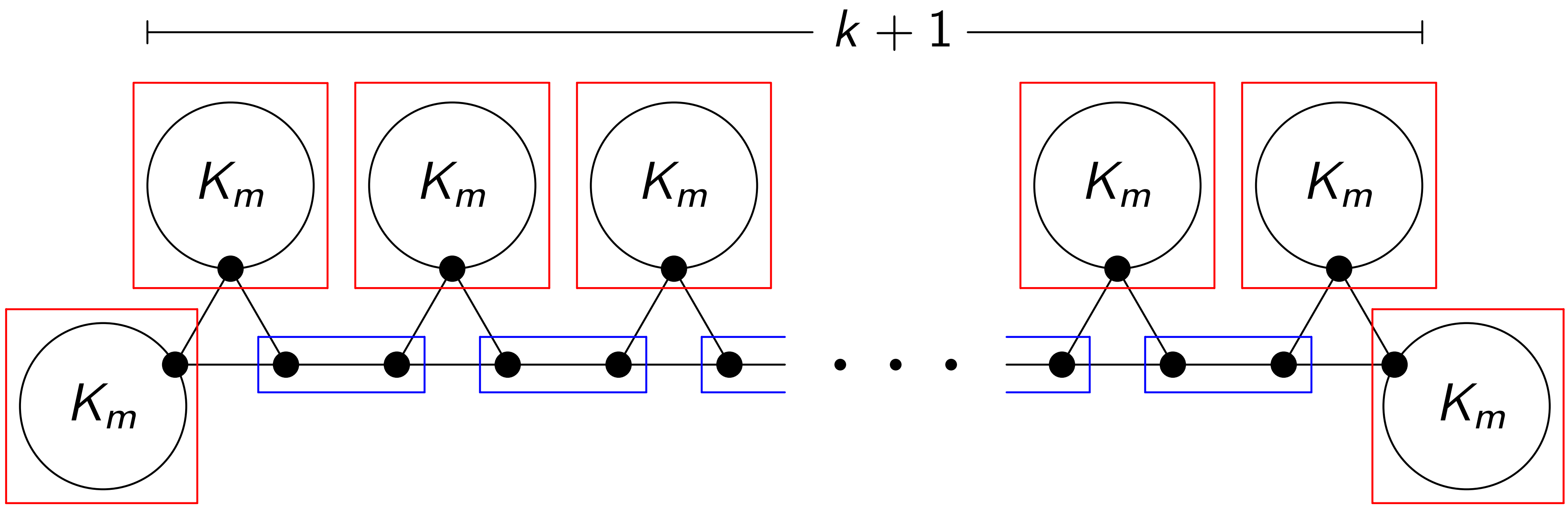
$$|X| \leq k+3$$



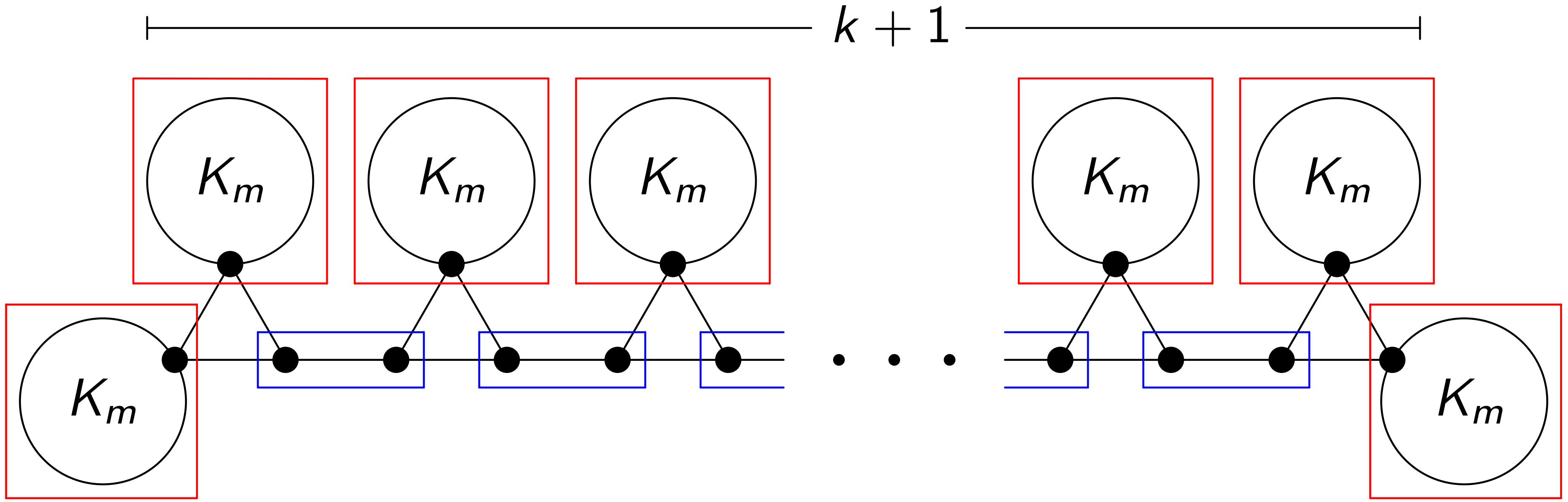
$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| \leq k + 3 + k$$



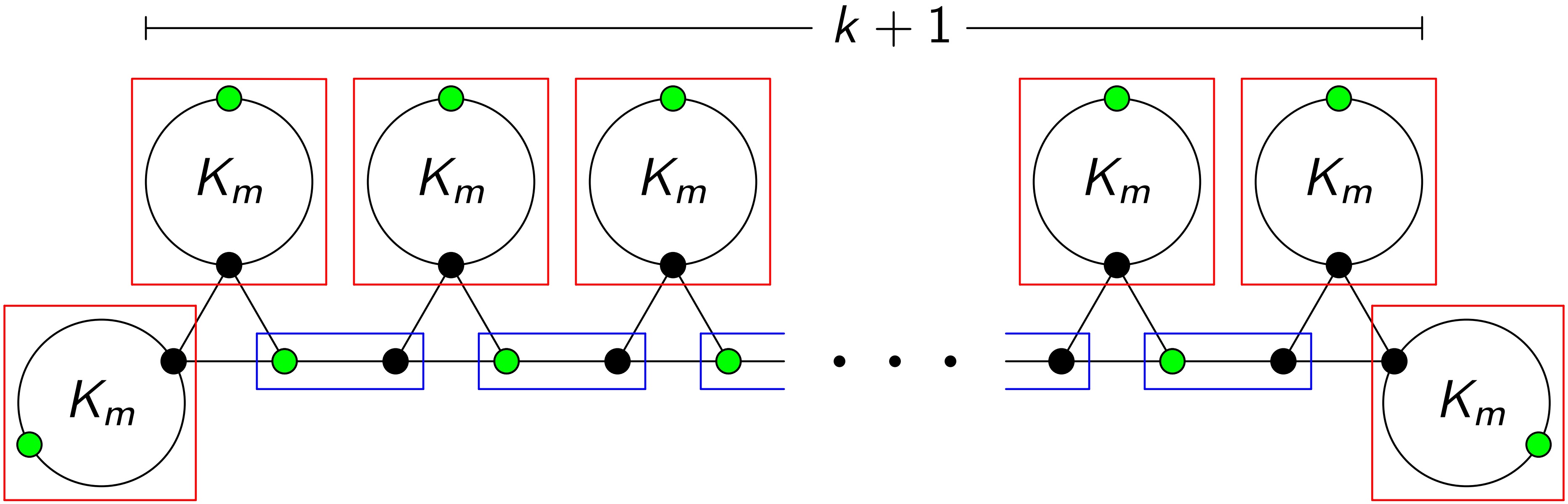
$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| \leq k + 3 + k = 2k + 3$$



$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| = k + 3 + k = 2k + 3$$



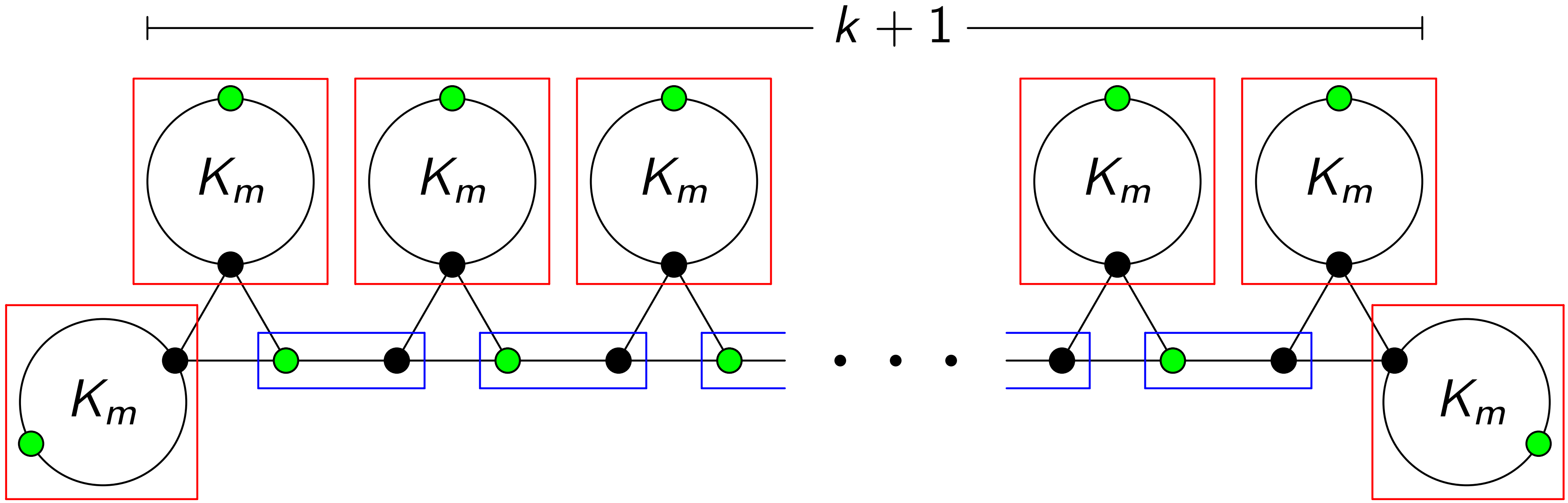
$$|V(G)| = m(k+3) + 2k = mk + 3m + 2k$$
$$|X| = k + 3 + k = 2k + 3$$
$$\sum_{x \in X} \deg(x)$$



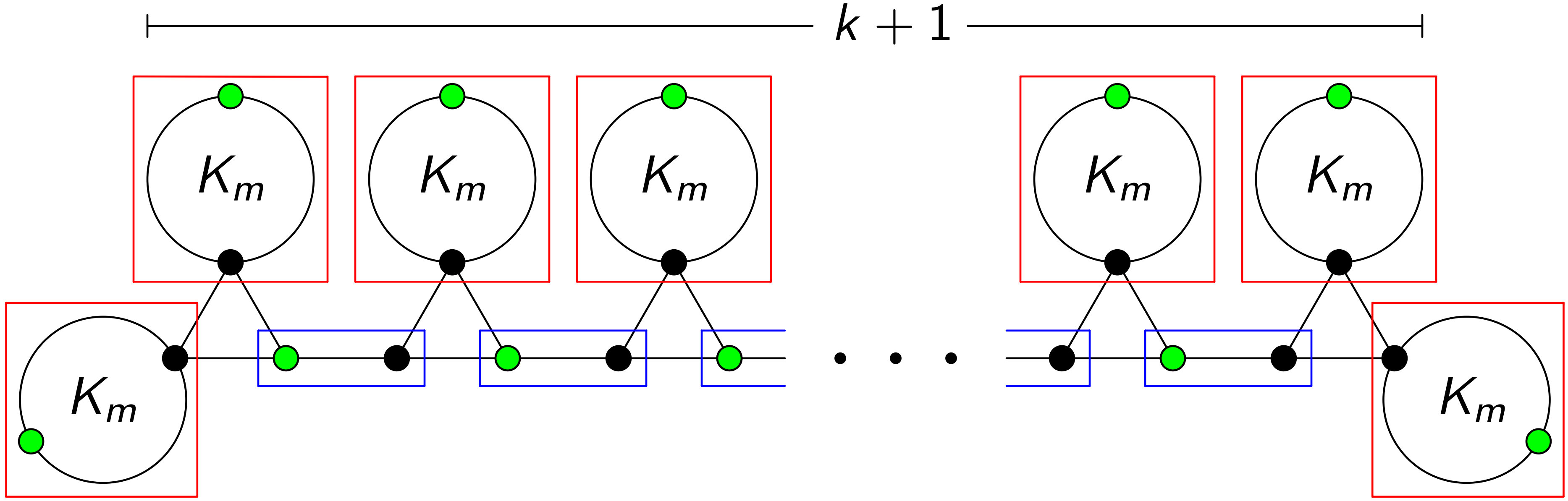
$$|V(G)| \quad = \quad m(k+3) + 2k = mk + 3m + 2k$$

$$|X| \quad = \quad k + 3 + k = 2k + 3$$

$$\sum_{x \in X} \deg(x) \qquad (k+3)(m \quad 1) + 3k$$



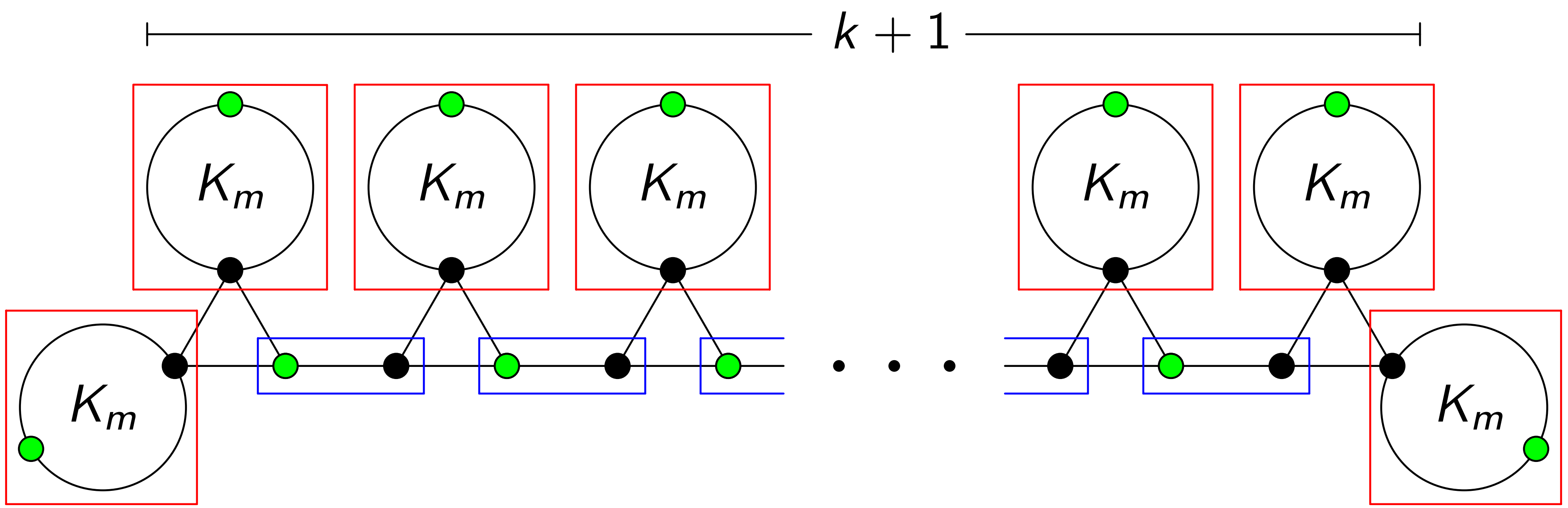
$$
\begin{aligned}
|V(G)| \quad &= \quad m(k+3) + 2k = mk + 3m + 2k \\
|X| \quad &= \quad k + 3 + k = 2k + 3 \\
\sum_{x \in X} \deg(x) \quad & \quad\ (k+3)(m \quad 1) + 3k \\
&= \quad mk \quad k + 3m \quad 3 + 3k \\
&= \quad mk + 3m + 2k \quad 3 = |V(G)| \quad 3
\end{aligned}
$$



$|X| \quad = \qquad 2k+3$

$\sum_{x \in X} \deg(x)$

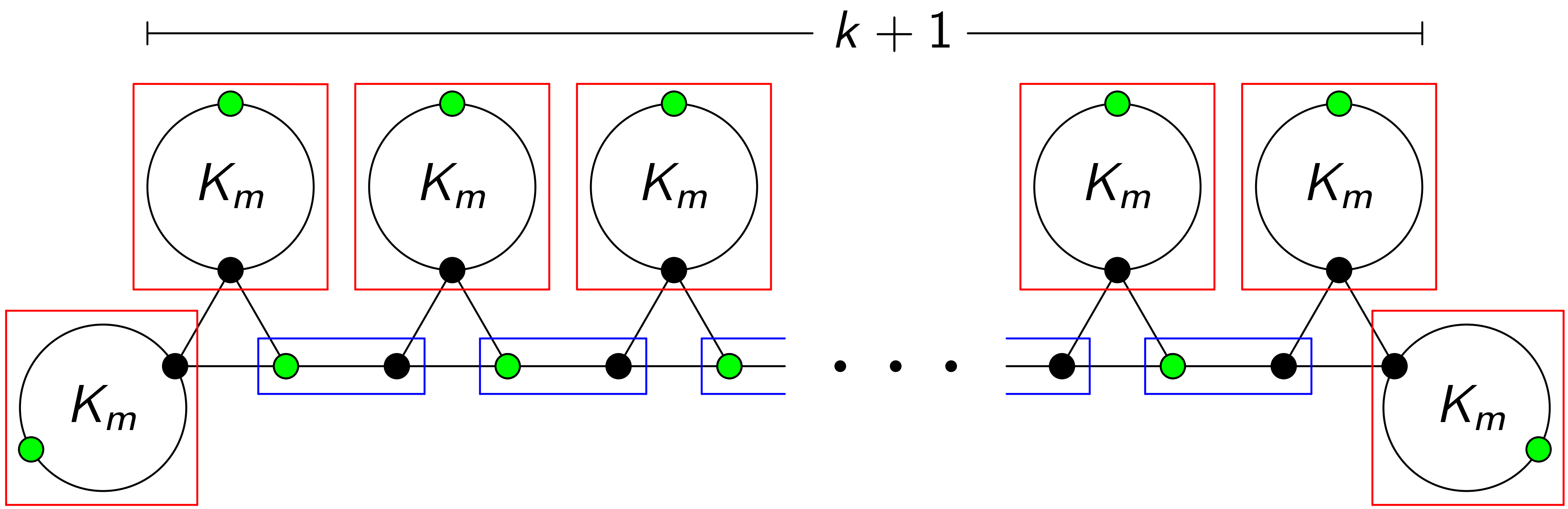
$|V(G)| \quad 3$



$$|X| \quad = \quad 2k+3$$

$$\sum_{x \in X} \deg(x) \qquad |V(G)| \quad 3$$

Goal: Guarantee a spanning tree with at most ______ branch vertices or a ______-vertex independent set with total degree at most ______



$$|X| = 2k+3$$

$$\sum_{x \in X} \deg(x) \qquad |V(G)| \quad 3$$

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)| \quad 3$

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set (with total degree at most $|V(G)|$ 3)

	Claw-free	Chair-free
non-total		
yes-total		

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set (with total degree at most $|V(G)|$ 3)

	Claw-free	Chair-free
non-total	M.O.Y. (2014)	
yes-total	Gould, Shull (2020)	

Theorem (Gould, Shull 2020)

Let k be a non-negative integer and let G be a connected claw-free graph. Then G contains either a spanning tree with at most k branch vertices, or an independent set of $2k+3$ vertices with total degree at most $|V(G)|$ 3.

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set (with total degree at most $|V(G)|$ 3)

	Claw-free	Chair-free
non-total	M.O.Y. (2014)	Schrader, Shull (2025)
yes-total	Gould, Shull (2020)	

Theorem (Schrader, Shull 2025)

Let k be a non-negative integer and let G be a connected chair-free graph. Then G contains either a spanning tree with at most k branch vertices, or an independent set of $2k+3$ vertices.

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set (with total degree at most $|V(G)|$ 3)

	Claw-free	Chair-free
non-total	M.O.Y. (2014)	Schrader, Shull (2025)
yes-total	Gould, Shull (2020)	?

Total Degree Question		
	Claw-free	Chair-free
$k=0$		
$k=1$		
k 2		

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)|$ 3

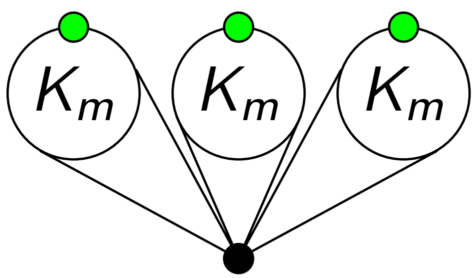
Total Degree Question		
	Claw-free	Chair-free
$k = 0$		
$k = 1$		
k 2		

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)|$ 3

Total Degree Question		
	Claw-free	Chair-free
$k = 0$	Kano, et. al. (2012)	
$k = 1$	M.O.Y. (2014)	
k 2	Gould, Shull (2020)	

Goal: Guarantee a spanning tree with at most 0 branch vertices or a $2(0)+3$-vertex independent set with total degree at most $|V(G)|\quad 3$

Total Degree Question		
	Claw-free	Chair-free
$k = 0$	Kano, et. al. (2012)	FALSE
$k = 1$	M.O.Y. (2014)	
$k \quad 2$	Gould, Shull (2020)	



Chair-free but NOT claw-free

$$|V(G)| = 3m+1$$
$$|X| = 3$$
$$\sum_{x \in X} \deg(x) \quad 3m$$
$$= |V(G)| \quad 1$$

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)| \quad 3$

Total Degree Question		
	Claw-free	Chair-free
$k = 0$	Kano, et. al. (2012)	FALSE
$k = 1$	M.O.Y. (2014)	?
$k \quad 2$	Gould, Shull (2020)	Future work

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)|$ 3

Total Degree Question		
	Claw-free	Chair-free
$k = 0$	Kano, et. al. (2012)	FALSE
$k = 1$	M.O.Y. (2014)	?
k 2	Gould, Shull (2020)	Future work

We use proof by contradiction. If there is no spanning tree with 1 branch vertex, we take the "best possible" spanning tree. This is the one with the fewest branch vertices, and in case of a tie, the fewest leaves.

Goal: Guarantee a spanning tree with at most k branch vertices or a $2k+3$-vertex independent set with total degree at most $|V(G)|$ 3

Total Degree Question		
	Claw-free	Chair-free
$k=0$	Kano, et. al. (2012)	FALSE
$k=1$	M.O.Y. (2014)	?
k 2	Gould, Shull (2020)	Future work

We use proof by contradiction. If there is no spanning tree with 1 branch vertex, we take the "best possible" spanning tree. This is the one with the fewest branch vertices, and in case of a tie, the fewest leaves.

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4		
2	5		
2	6		
3	5		

Goal: Guarantee a spanning tree with at most 1 branch vertices or a $2(1)+3$-vertex independent set with total degree at most $|V(G)|$ 3

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4		
2	5		
2	6		
3	5		

Goal: Guarantee a spanning tree with at most 1 branch vertex or a 5-vertex independent set with total degree at most $|V(G)|$ 3

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4		
2	5	Kano, et. al. (2012)	
2	6	Kano, et. al. (2012)	
3	5	Kano, et. al. (2012)	

Theorem (Kano, et. al. 2012)

Let h be a non-negative integer and let G be a connected claw-free graph. Then G contains either a spanning tree with at most $h+2$ leaves, or an independent set of $h+3$ vertices with total degree at most $|V(G)|$ h 3.

Corollary ($h = 2$)

Let G be a connected claw-free graph. Then G contains either a spanning tree with at most 4 leaves, or an independent set of 5 vertices with total degree at most $|V(G)|$ 5.

Goal: Guarantee a spanning tree with at most 1 branch vertex or a 5-vertex independent set with total degree at most $|V(G)| \quad 3$

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4	M.O.Y. (2014)	
2	5	Kano, et. al. (2012)	
2	6	Kano, et. al. (2012)	
3	5	Kano, et. al. (2012)	

Theorem (Matsuda, Ozeki, Yamashita 2014)

Let G be a connected claw-free graph. Then G contains either a spanning tree with at most 1 branch vertex, or an independent set of 5 vertices with total degree at most $|V(G)| \quad 3$.

Goal: Guarantee a spanning tree with at most 1 branch vertex or a 5-vertex independent set with total degree at most $|V(G)|$ 3

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4	M.O.Y. (2014)	Schrader, Shull (2025)
2	5	Kano, et. al. (2012)	
2	6	Kano, et. al. (2012)	
3	5	Kano, et. al. (2012)	

Theorem (Schrader, Shull 2025)

Let G be a connected chair-free graph. *If G has a spanning tree with at most 4 leaves*, then G contains either a spanning tree with at most 1 branch vertex, or an independent set of 5 vertices with total degree at most $|V(G)|$ 3.

Goal: Guarantee a spanning tree with at most 1 branch vertex or a 5-vertex independent set with total degree at most $|V(G)|$ 3

k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4	M.O.Y. (2014)	Schrader, Shull (2025)
2	5	Kano, et. al. (2012)	OUR RESULT
2	6	Kano, et. al. (2012)	Future work
3	5	Kano, et. al. (2012)	Future work

Goal: Guarantee a spanning tree with at most 1 branch vertex or a 5-vertex independent set with total degree at most $|V(G)|$ 3

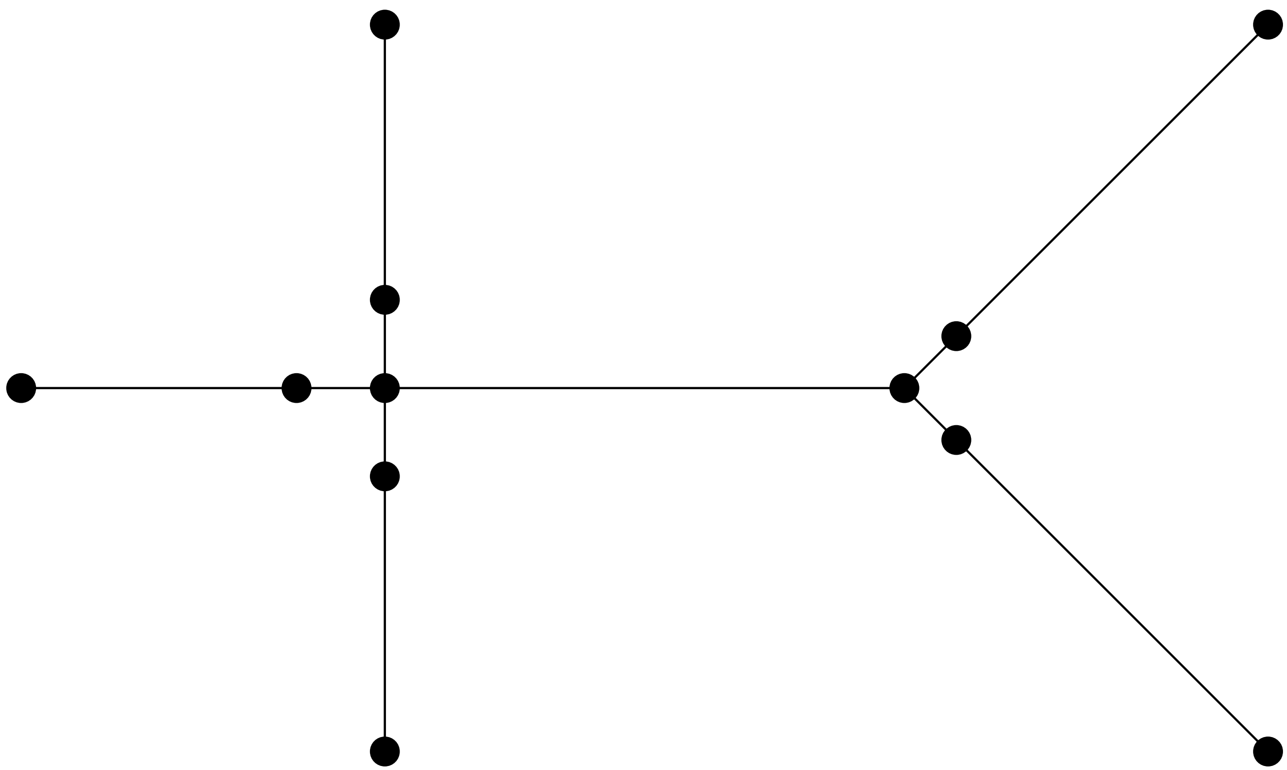
k = 1 best possible tree			
Branch vertices	Leaves	Claw-free	Chair-free
2	4	M.O.Y. (2014)	Schrader, Shull (2025)
2	5	Kano, et. al. (2012)	OUR RESULT
2	6	Kano, et. al. (2012)	Future work
3	5	Kano, et. al. (2012)	Future work

Theorem (B., Shull)

Let G be a connected chair-free graph. If G has a spanning tree with at most 5 leaves and 2 branch vertices, then G contains either a spanning tree with at most 1 branch vertex or an independent set of 5 vertices with total degree at most $|V(G)|$ 3.

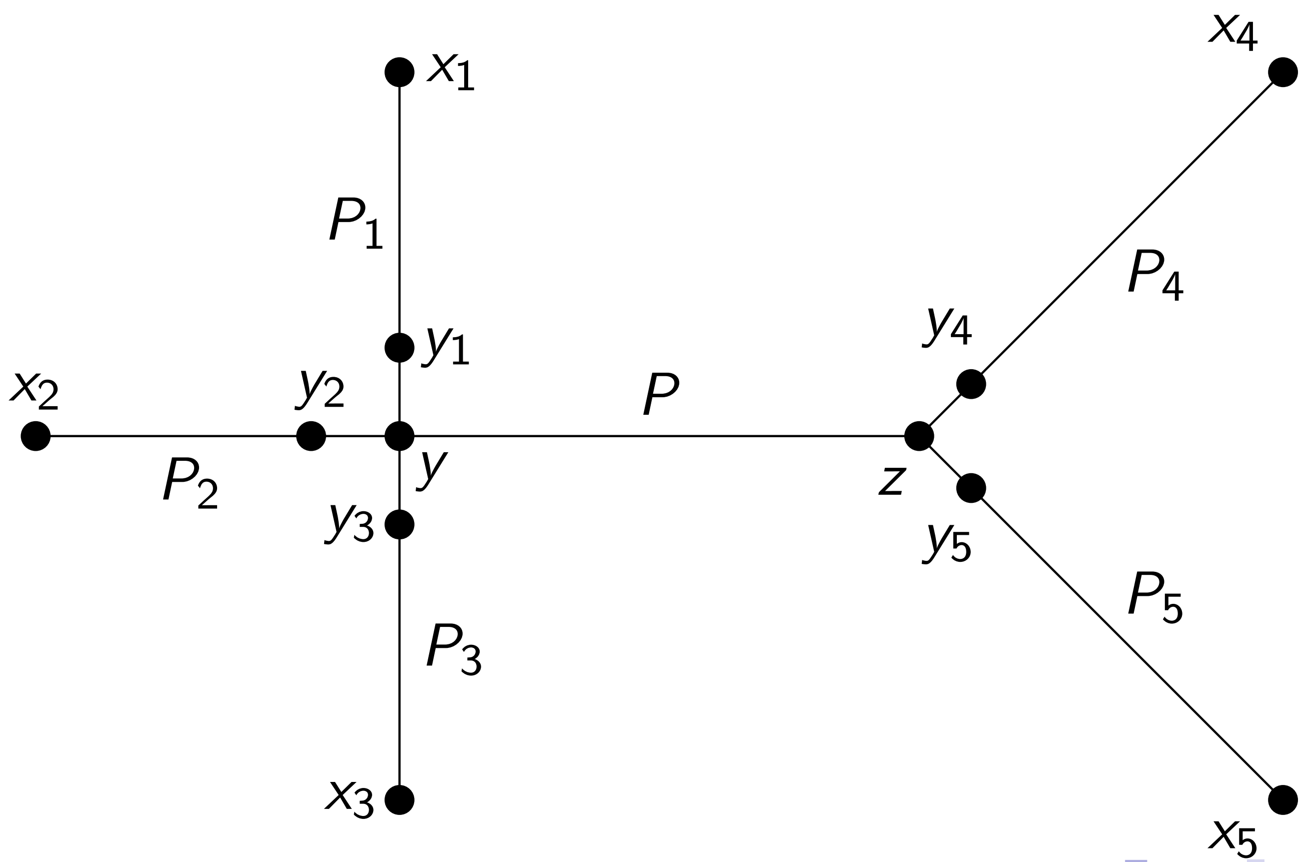
Theorem (B., Shull)

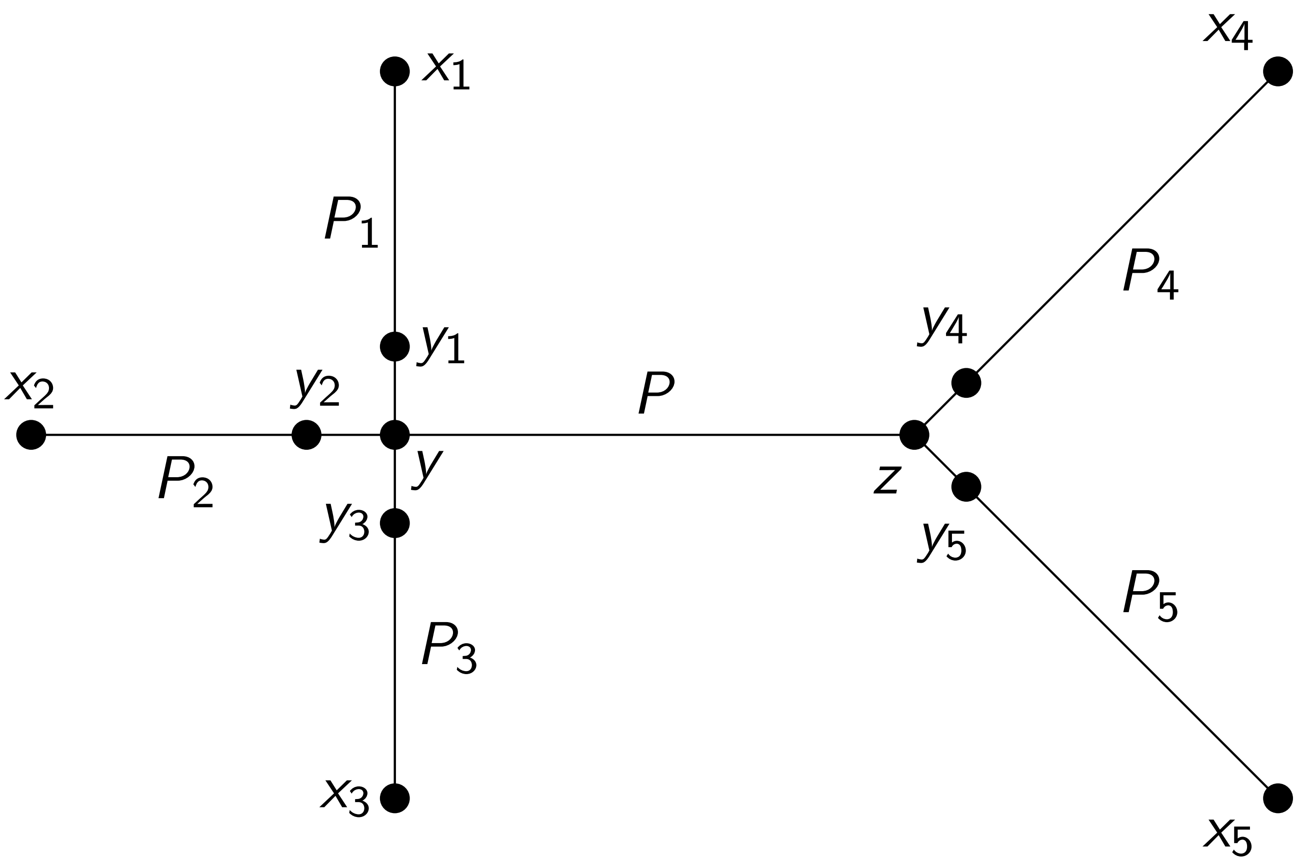
Let G be a connected chair-free graph. If G has a spanning tree with at most 5 leaves and 2 branch vertices, then G contains either a spanning tree with at most 1 branch vertex or an independent set of 5 vertices with total degree at most $|V(G)| \quad 3$.



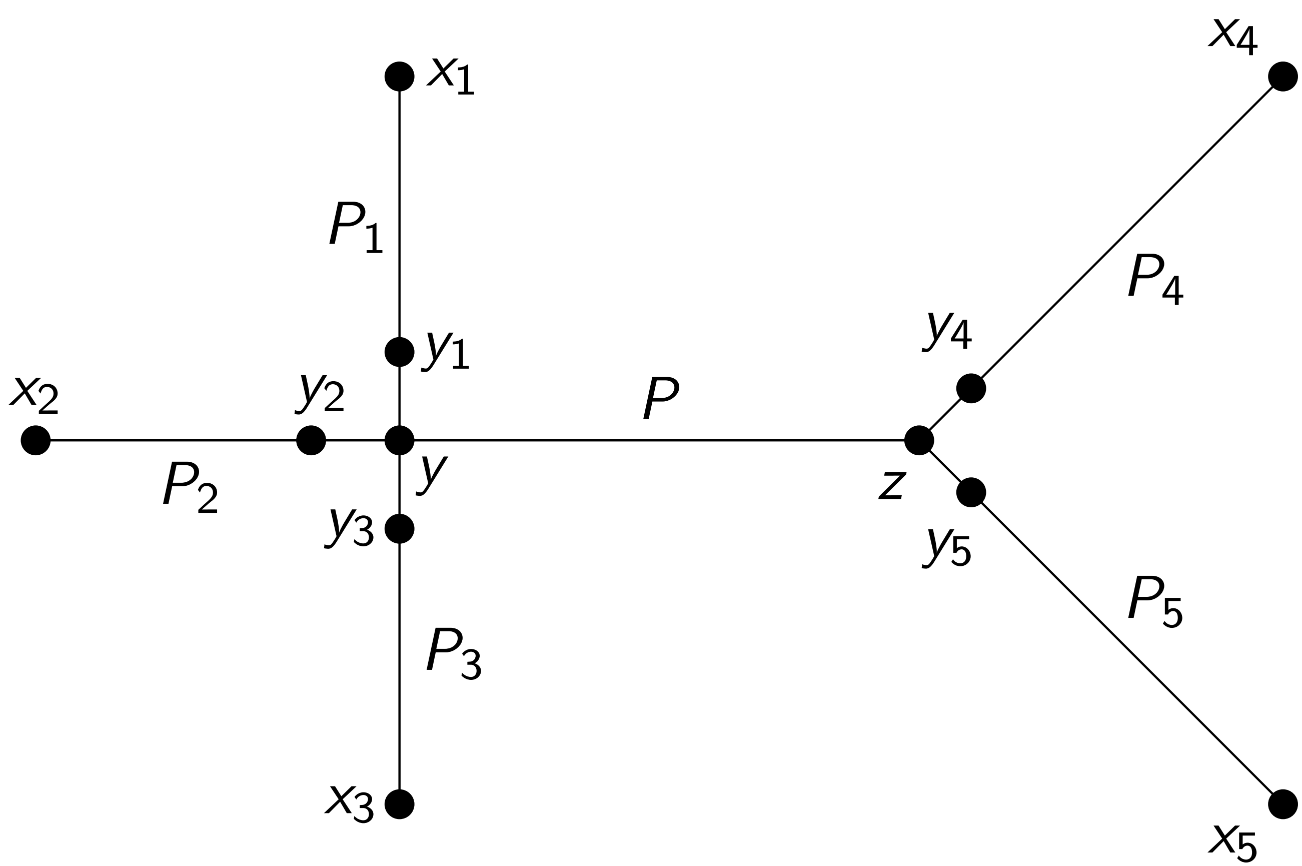
Theorem (B., Shull)

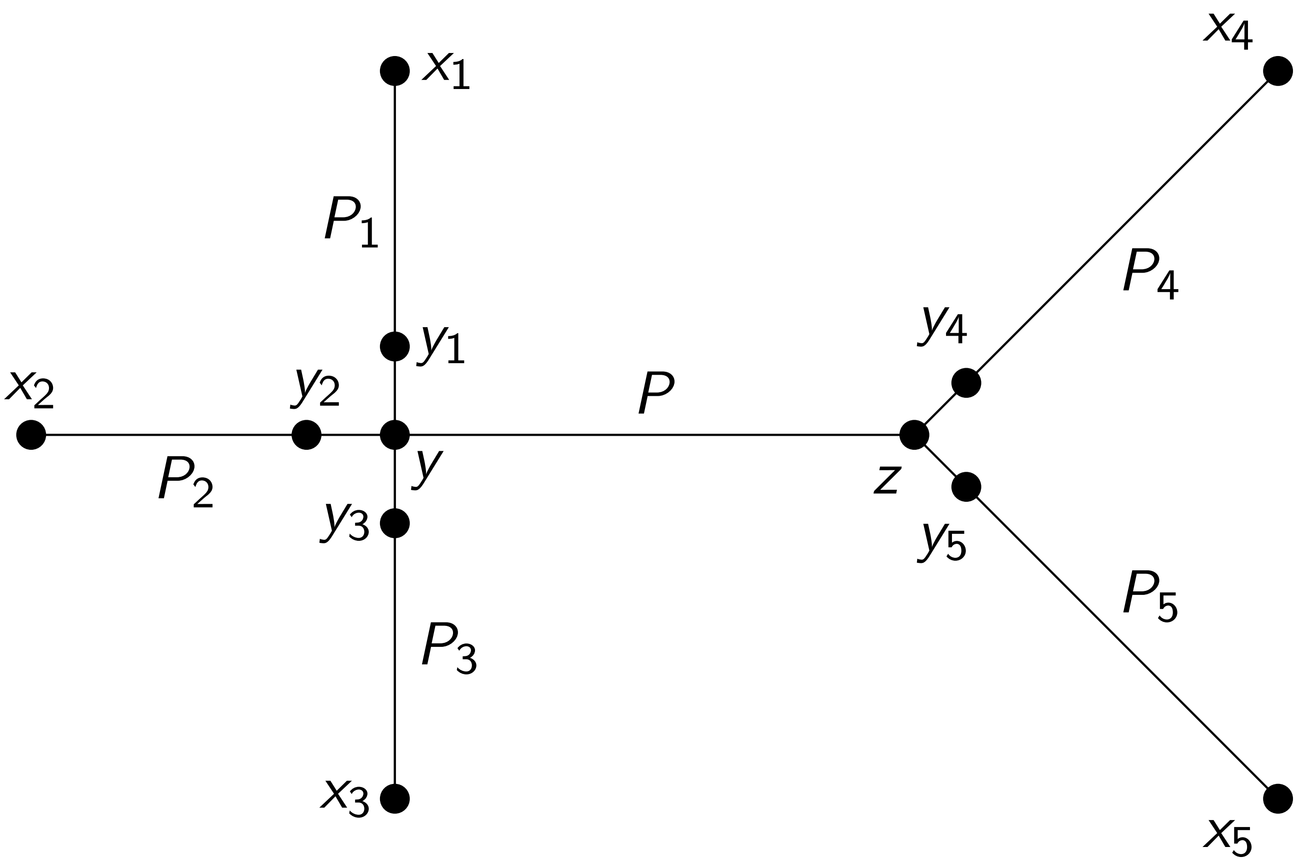
Let G be a connected chair-free graph. If G has a spanning tree with at most 5 leaves and 2 branch vertices, then G contains either a spanning tree with at most 1 branch vertex or an independent set of 5 vertices with total degree at most $|V(G)| \quad 3$.




x_1
P_1
y_1
x_2
y_2
P
y
P_2
y_3
P_3
x_3
x_4
P_4
y_4
z
y_5
P_5
x_5

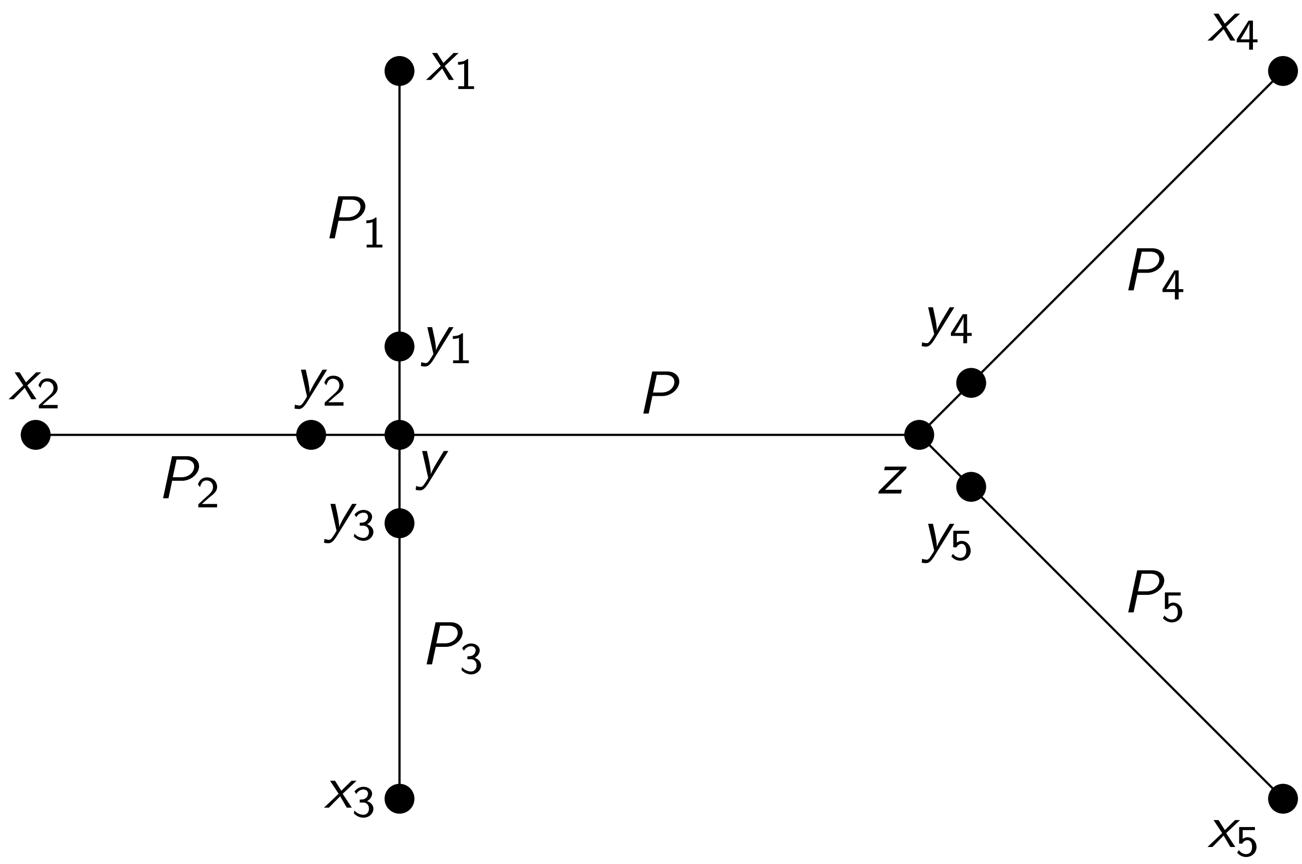
Choose the tree with the shortest possible P. If we find a tree with a shorter central path, fewer leaves, or fewer branch vertices, that is a contradiction.



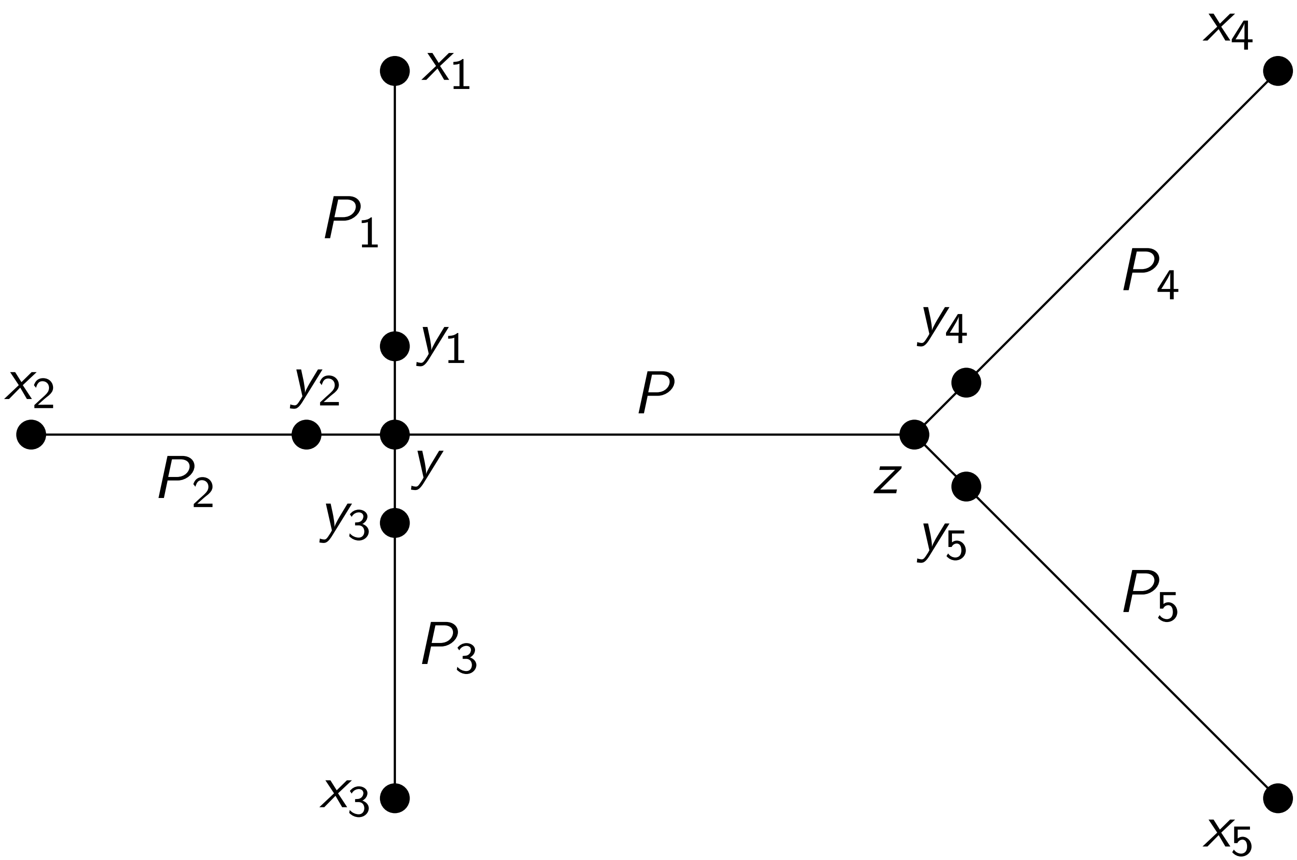


x_1
P_1
y_1
x_2
y_2
P_2
y
y_3
P_3
x_3
P
z
x_4
P_4
y_4
y_5
P_5
x_5

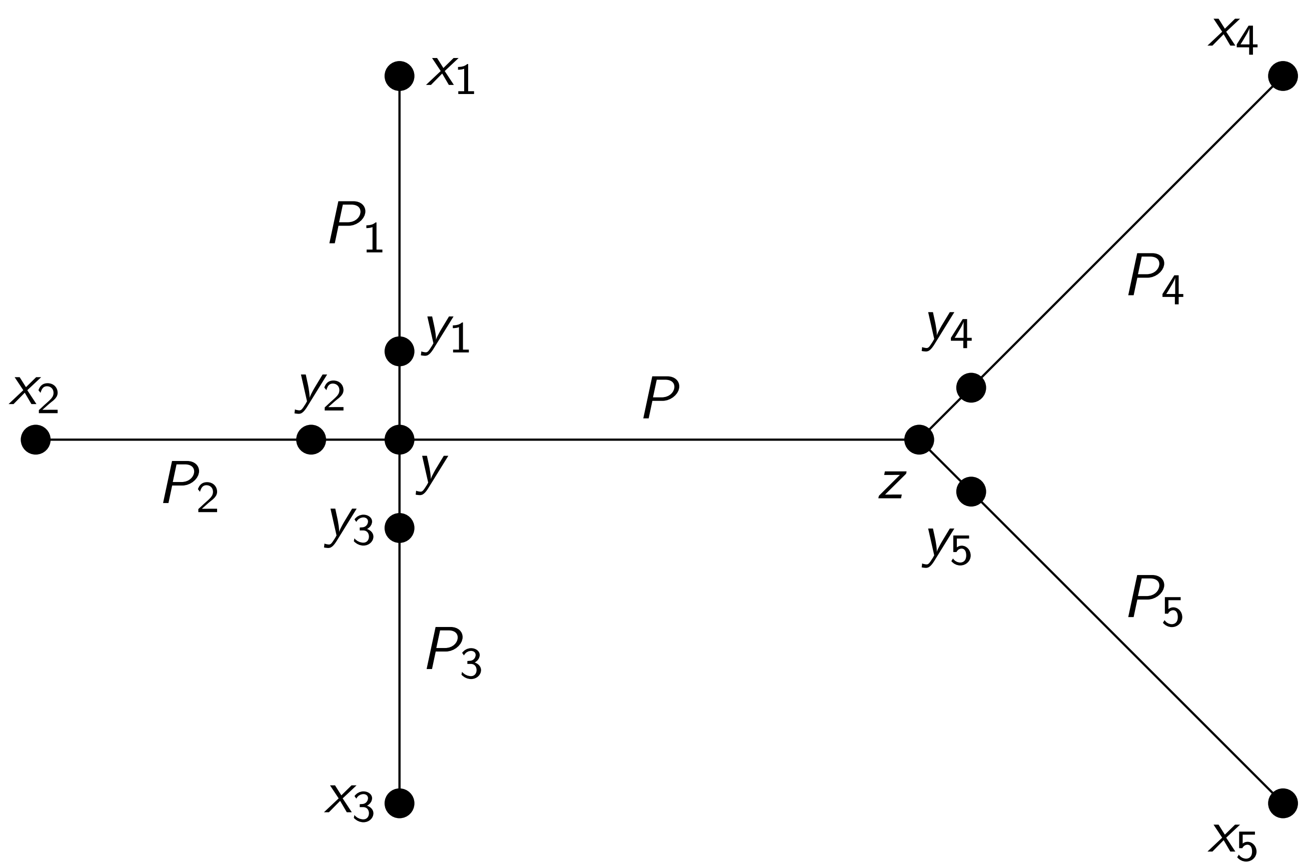
Claim: Either $|E(P)| = 1$ or $y_4 y_5 \in E(G)$.



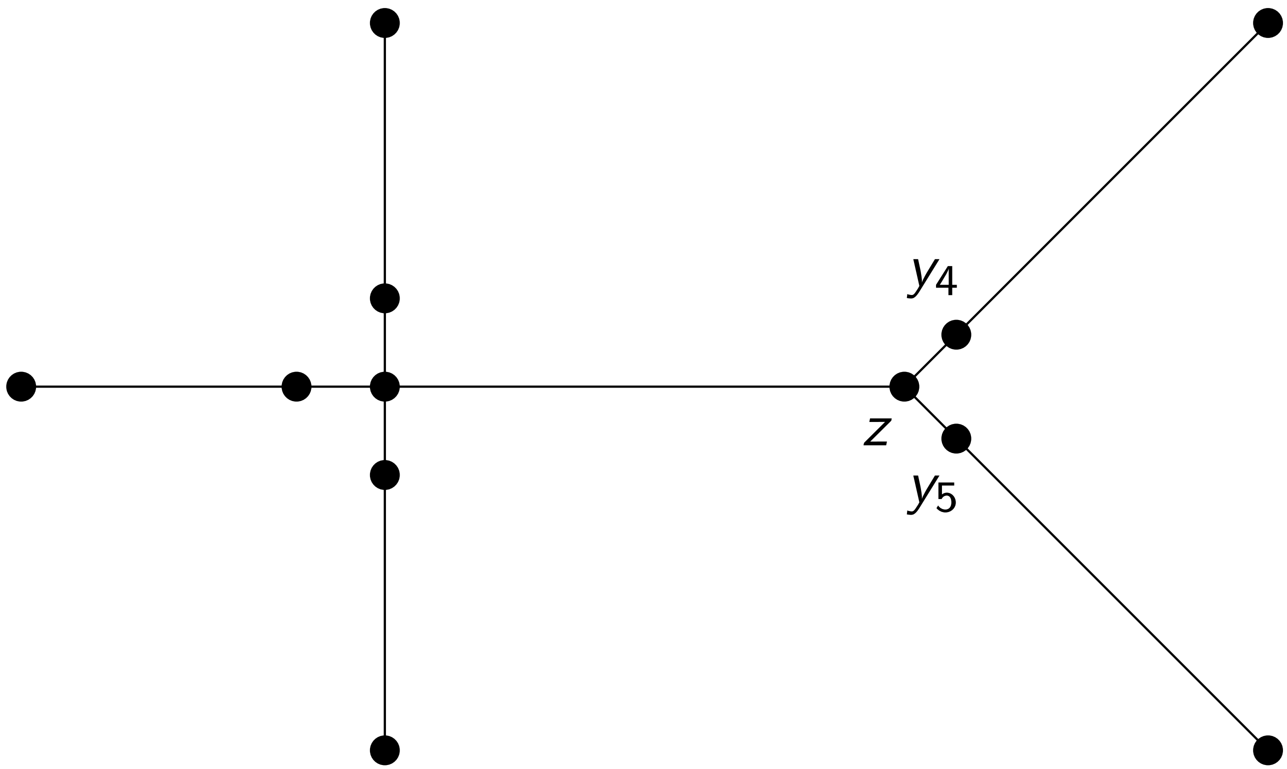
Claim: Either $|E(P)| = 1$ or $y_4 y_5 \in E(G)$. Proof by contradiction: Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



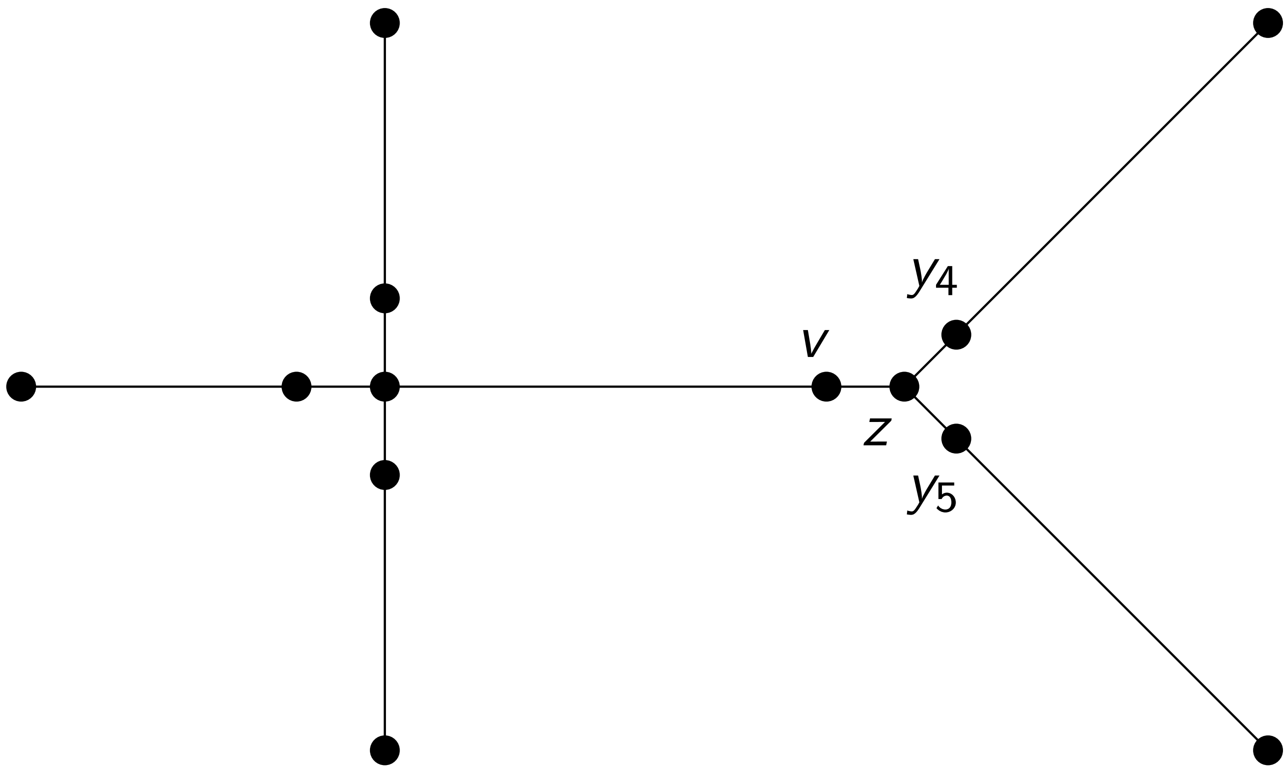
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



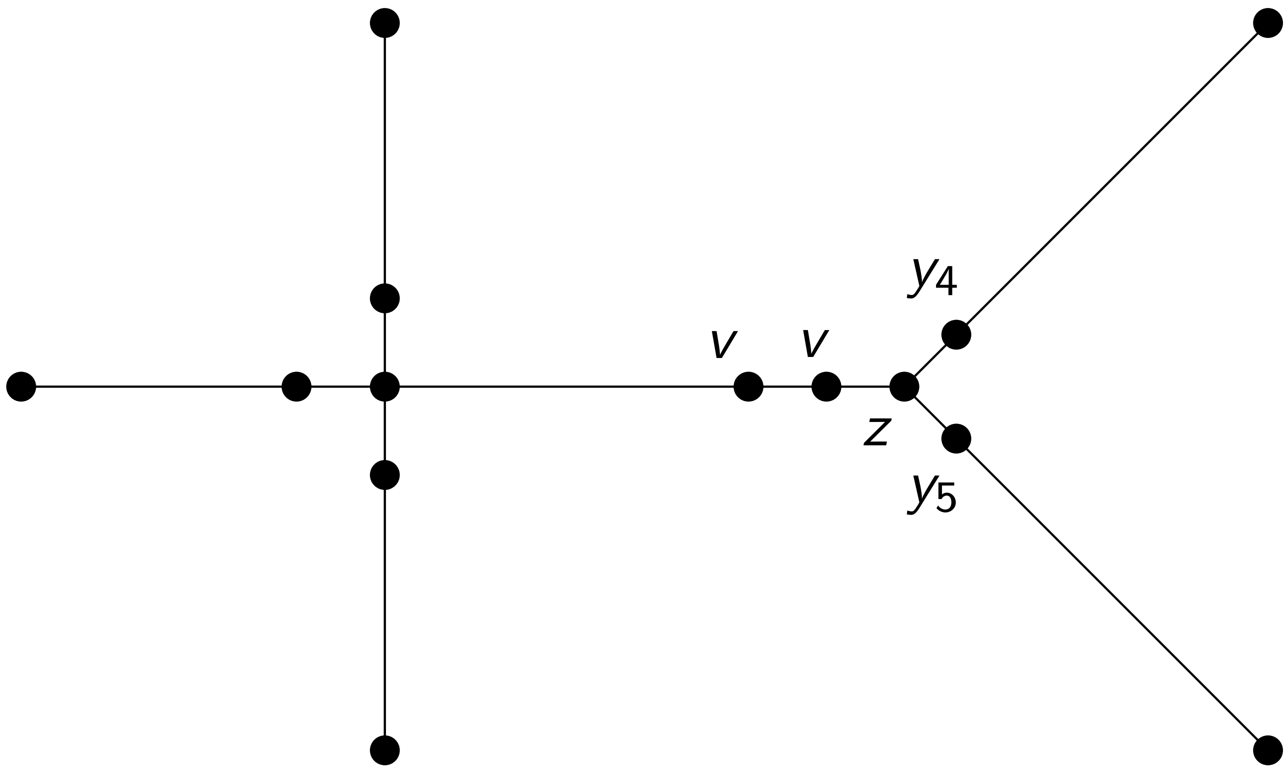
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

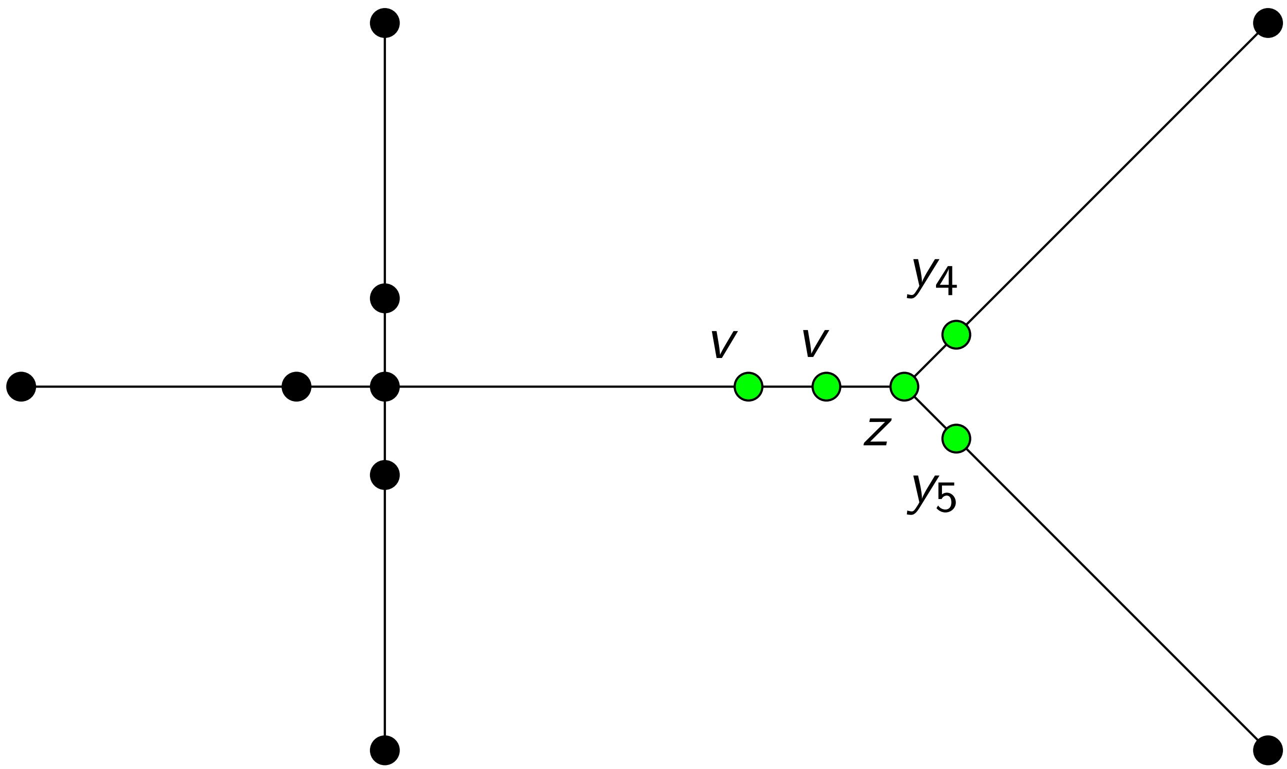


Assume $|E(P)| \quad 2$ and $y_4 y_5 \notin E(G)$.



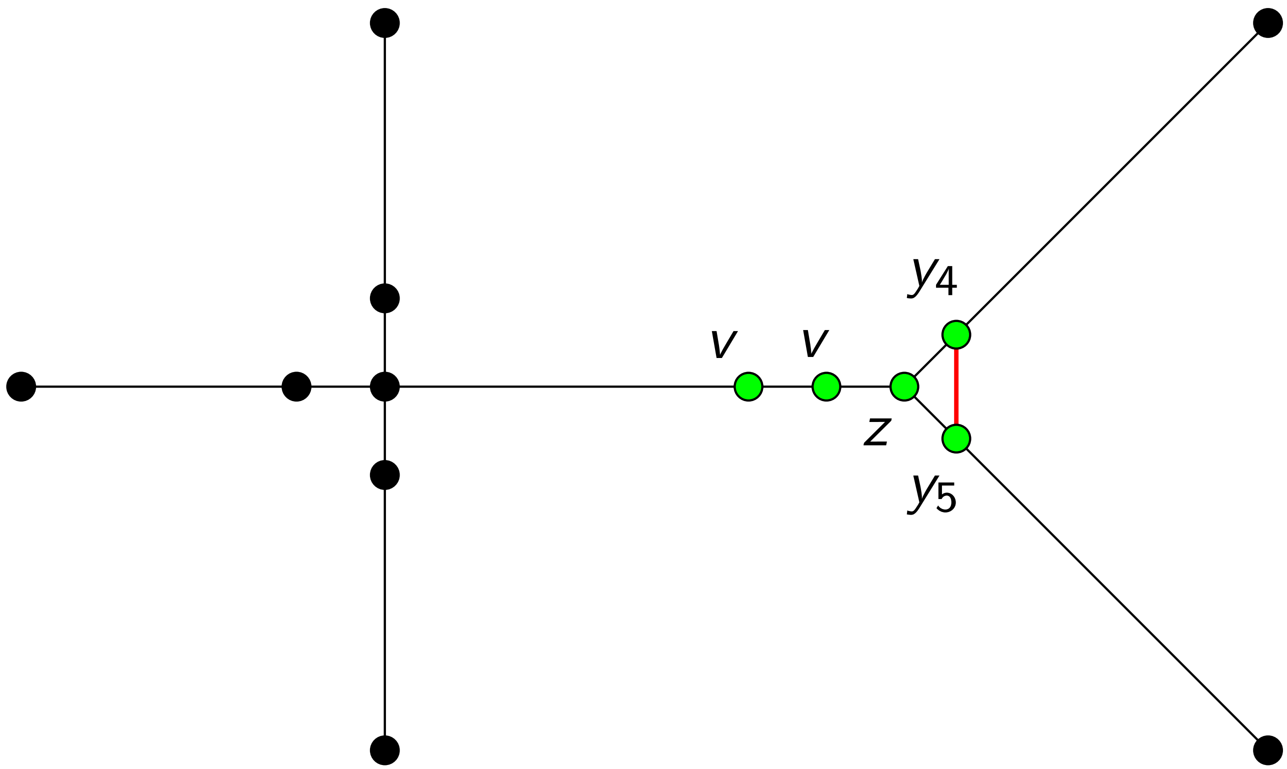
Assume $|E(P)|\quad 2$ and $y_4y_5 \notin E(G)$.

- y_4y_5, y_4v and y_5v, $y_4v\quad$ and $y_5v\quad$, zv



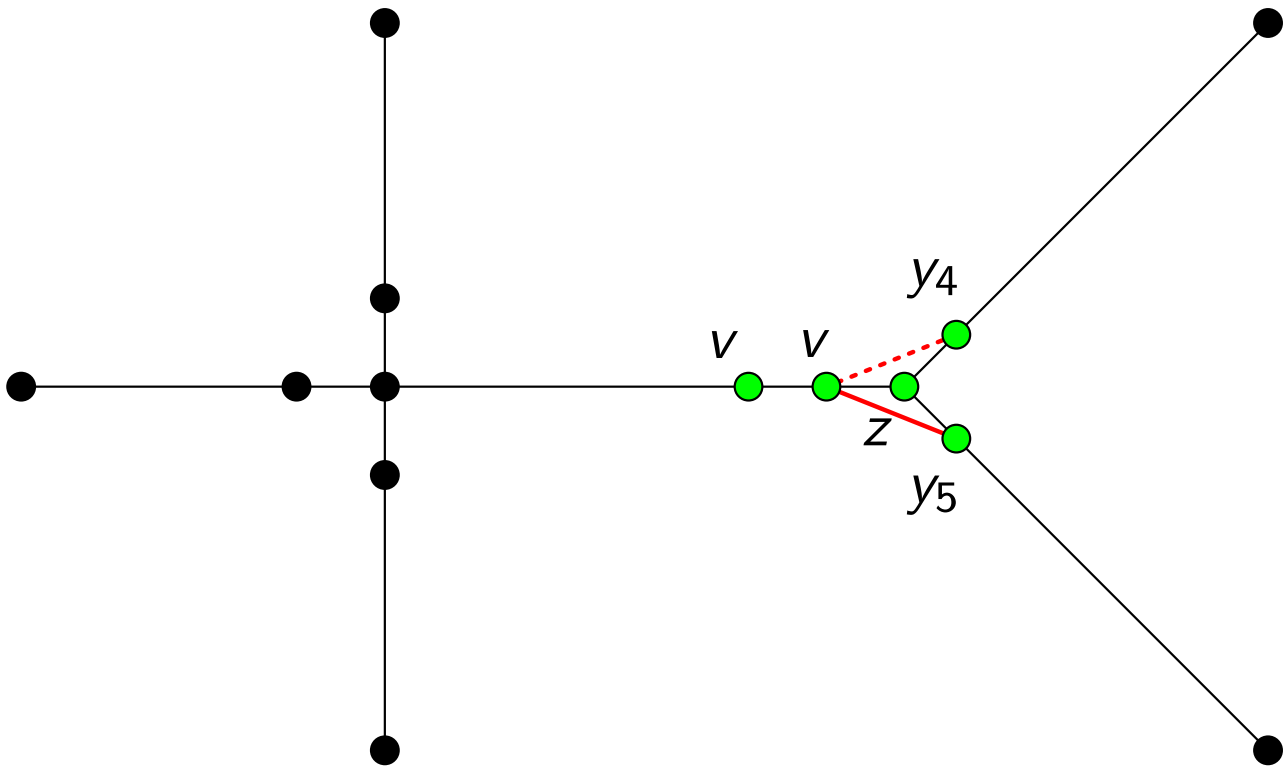
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

- y_4y_5, y_4v and y_5v, y_4v and y_5v , zv



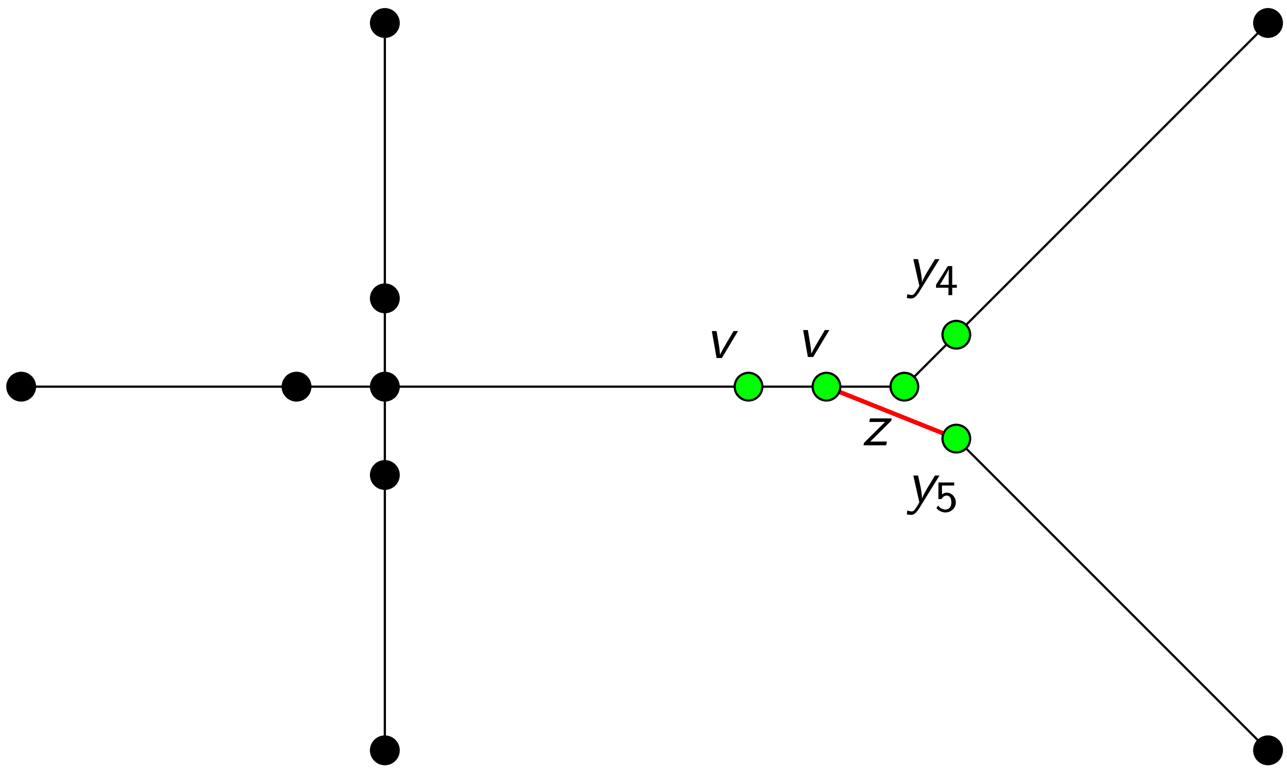
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, y_4v and y_5v, $y_4v \quad$ and $y_5v \quad$, zv



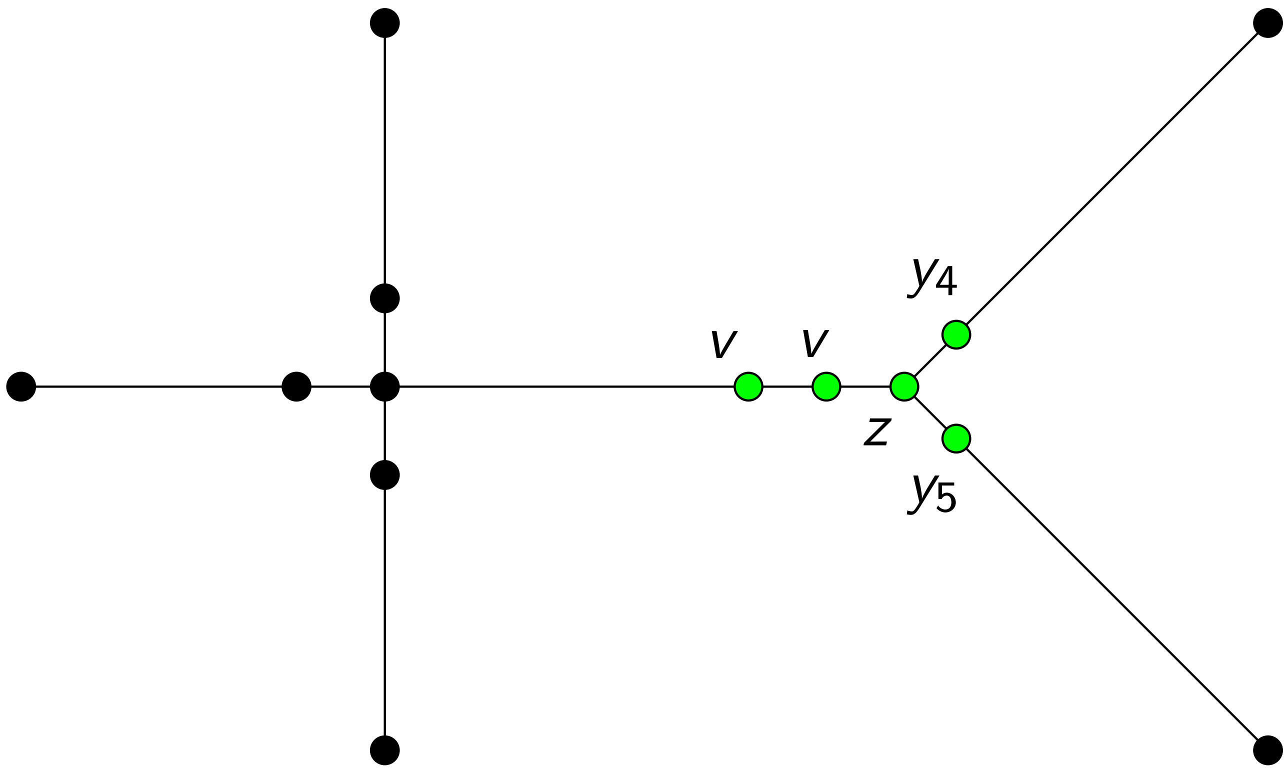
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, y_4v and y_5v, y_4v and y_5v , zv



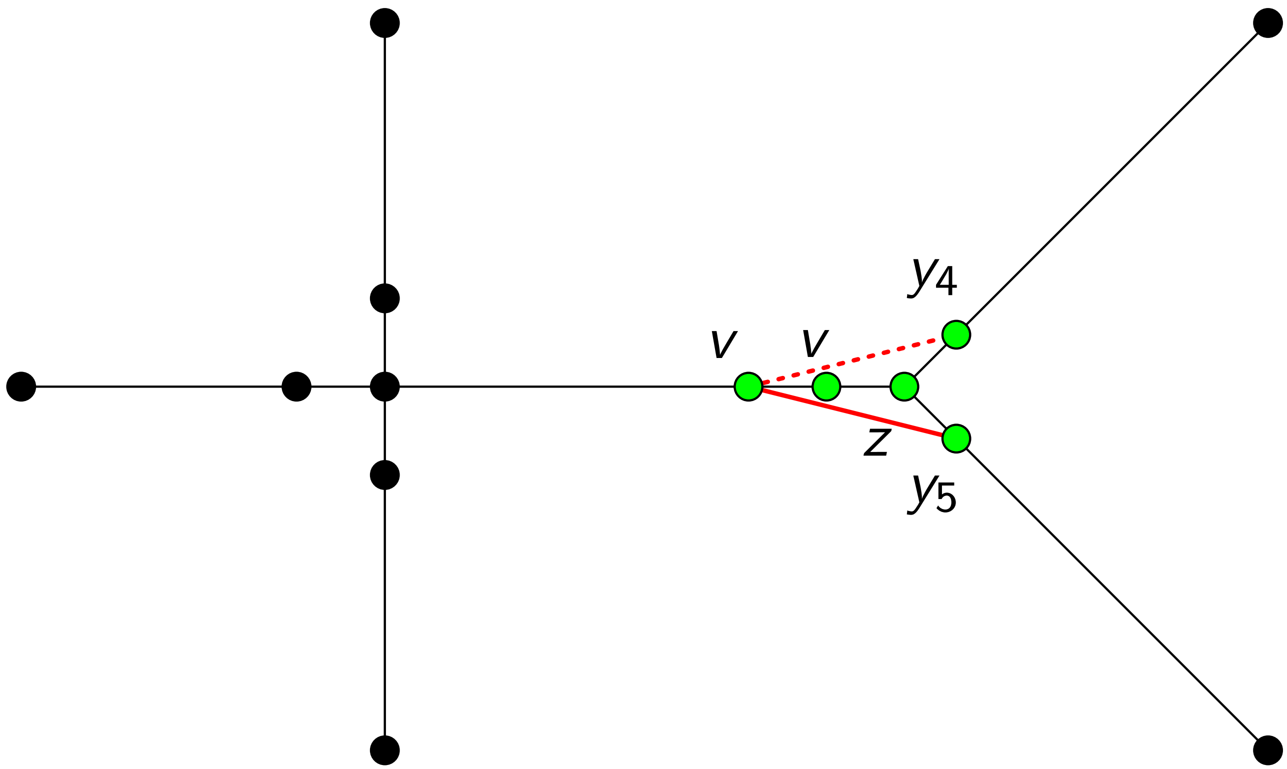
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4v and y_5v~~, y_4v and y_5v , zv



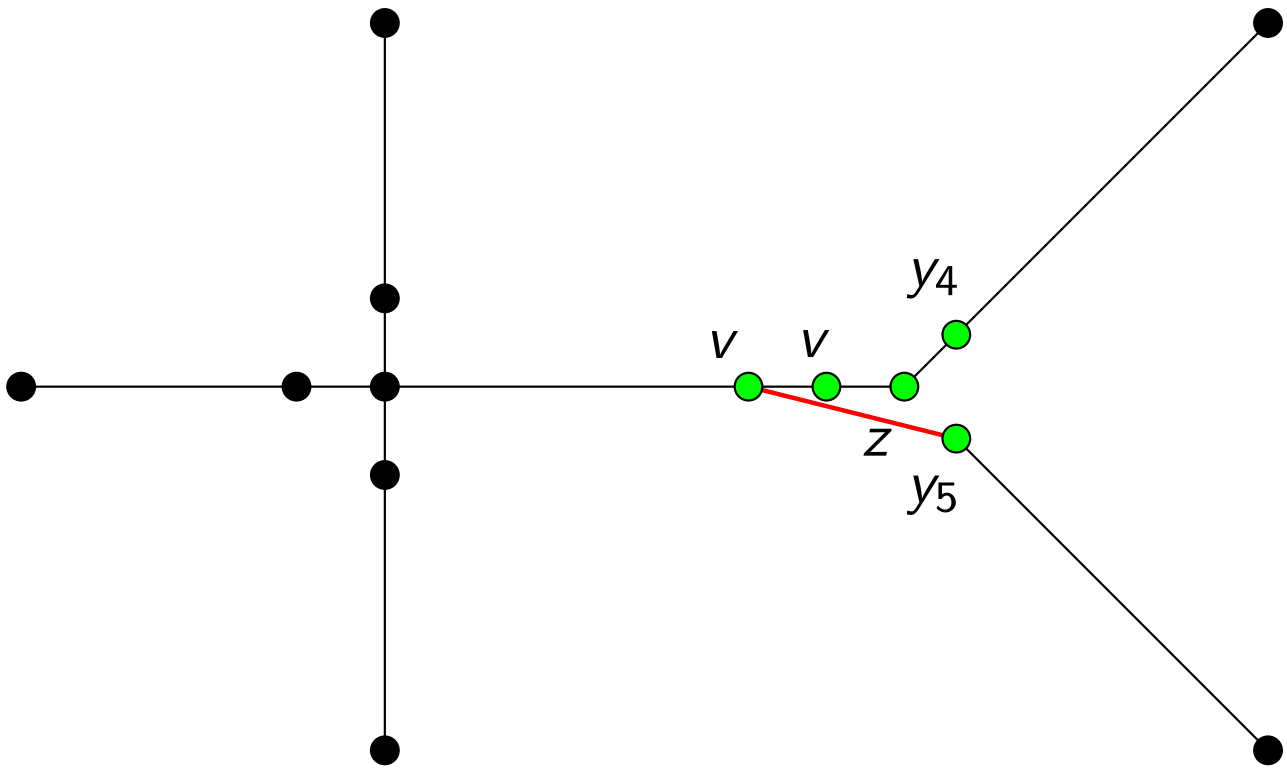
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, ~~$y_4 v$ and $y_5 v$~~, $y_4 v$ and $y_5 v$, zv



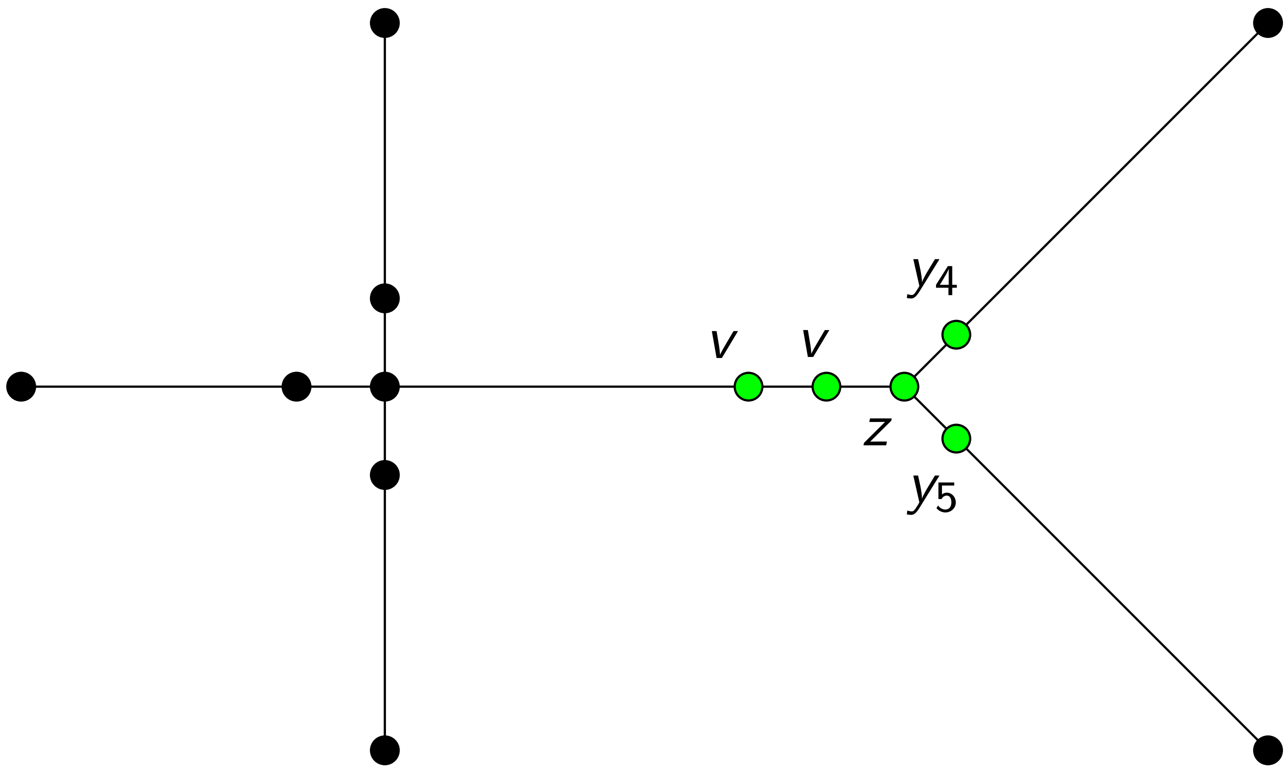
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4v and y_5v~~, y_4v and y_5v , zv



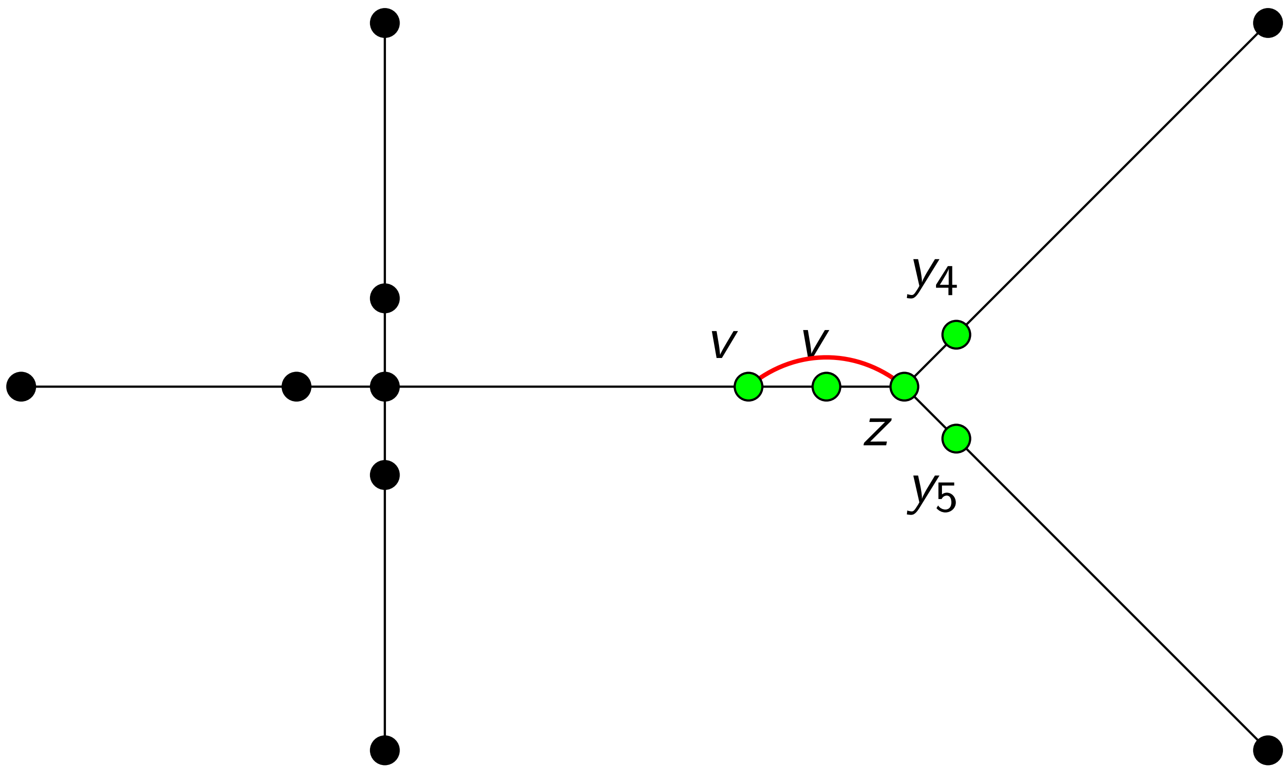
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4v and y_5v~~, ~~y_4v and y_5v~~, zv



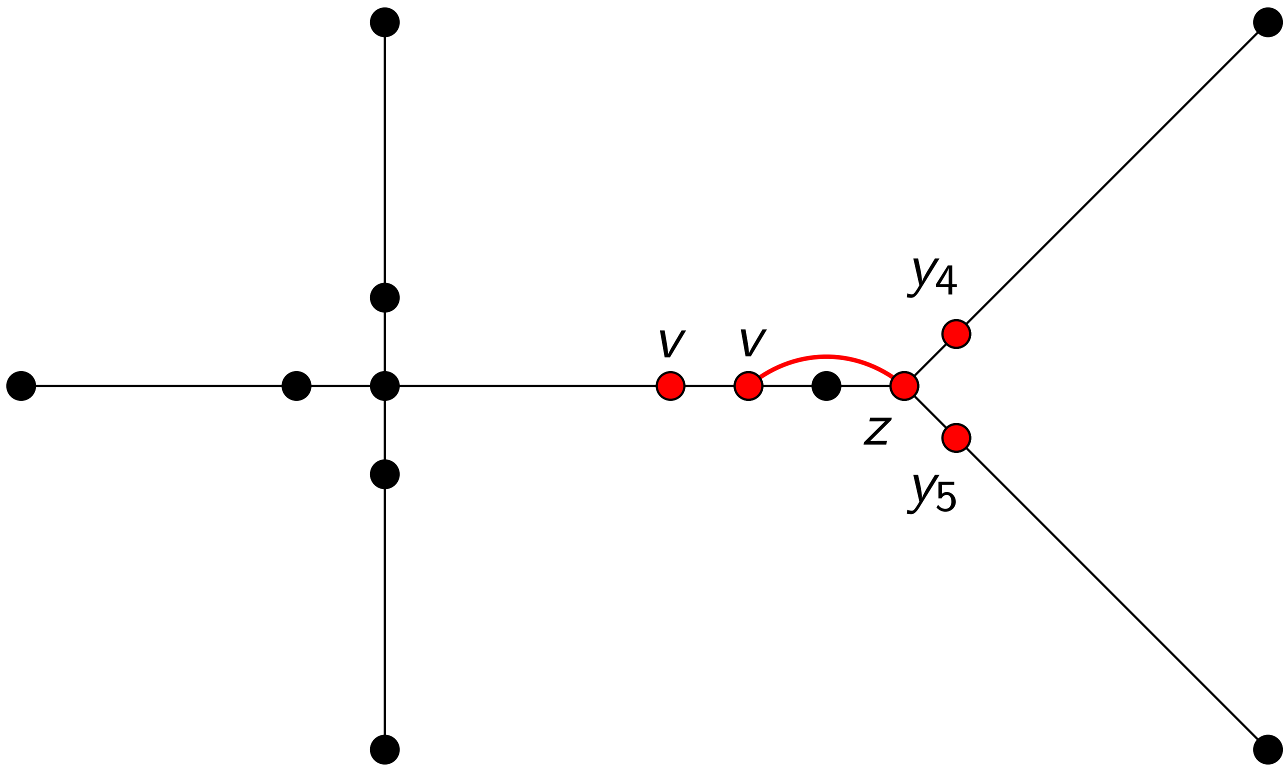
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4v and y_5v~~, ~~y_4v and y_5v~~, zv



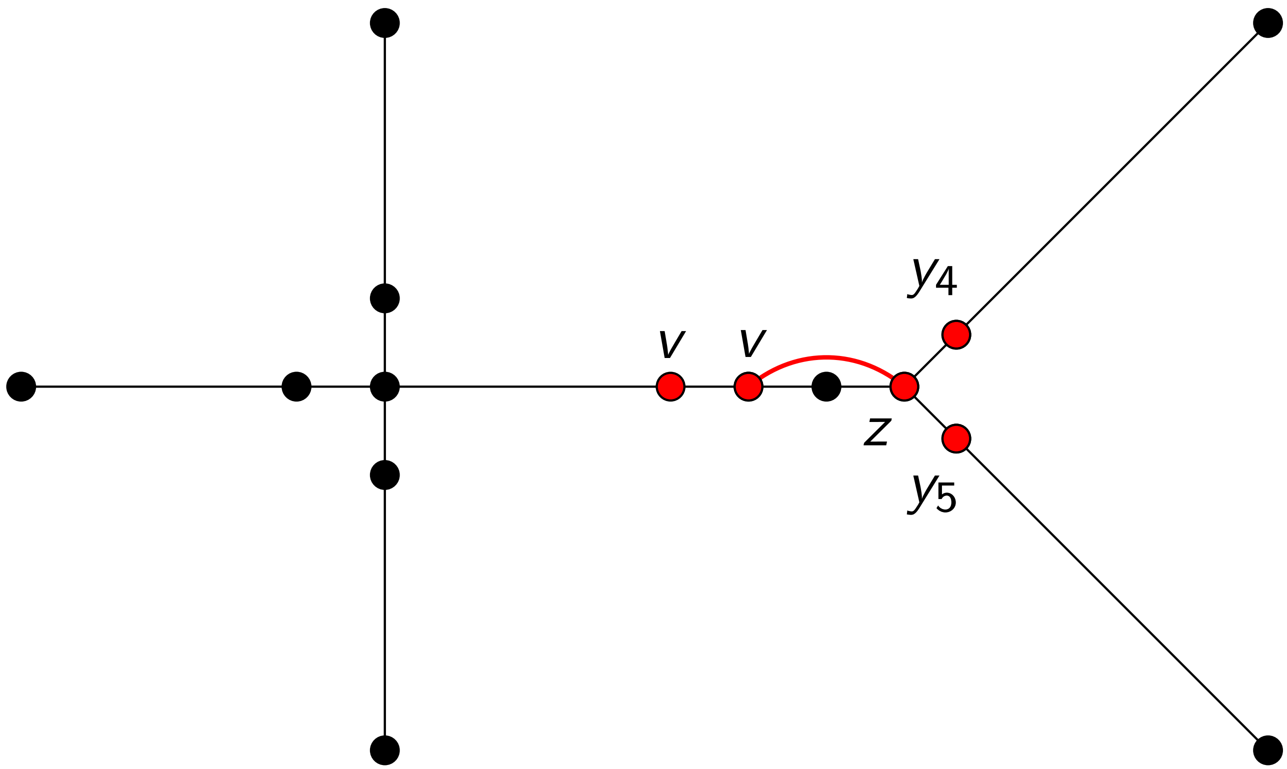
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

- Rename v to v and make a new chair! New edges $y_4 y_5$, $y_4 v$ and $y_5 v$, $y_4 v$ and $y_5 v$, zv.

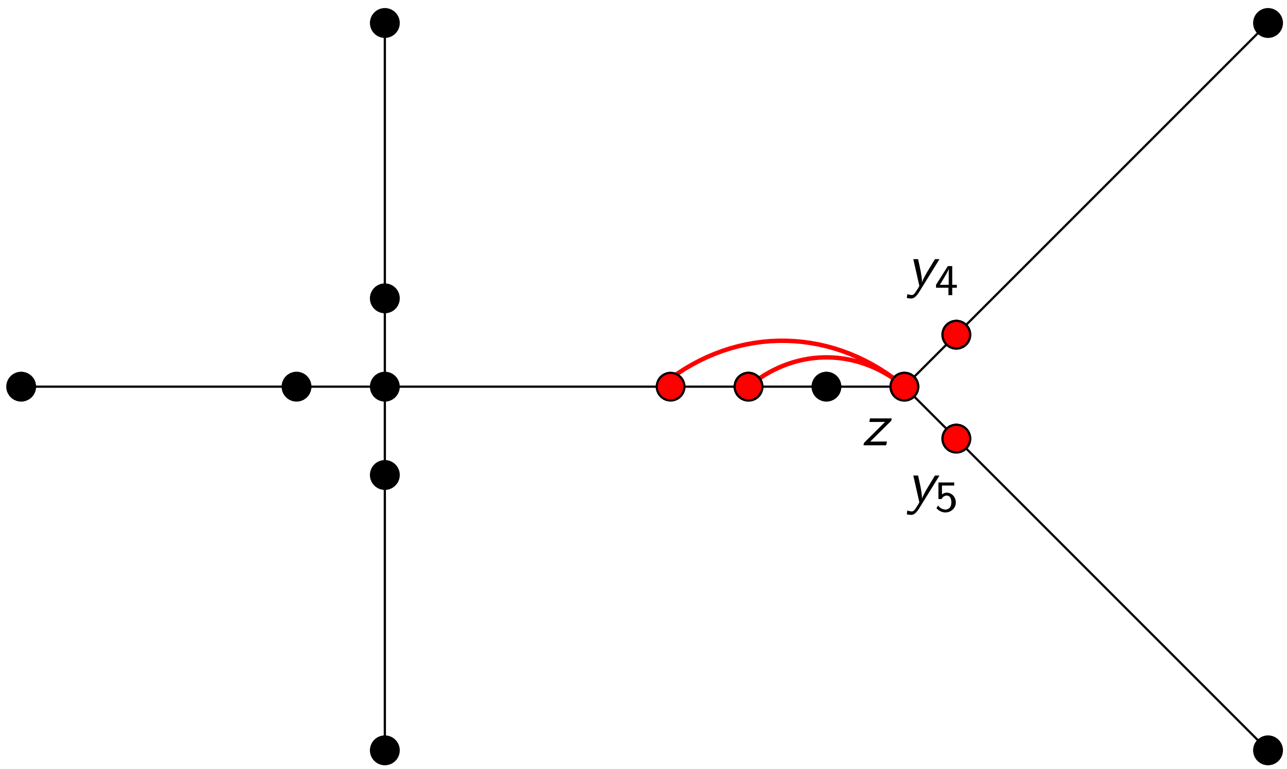


Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

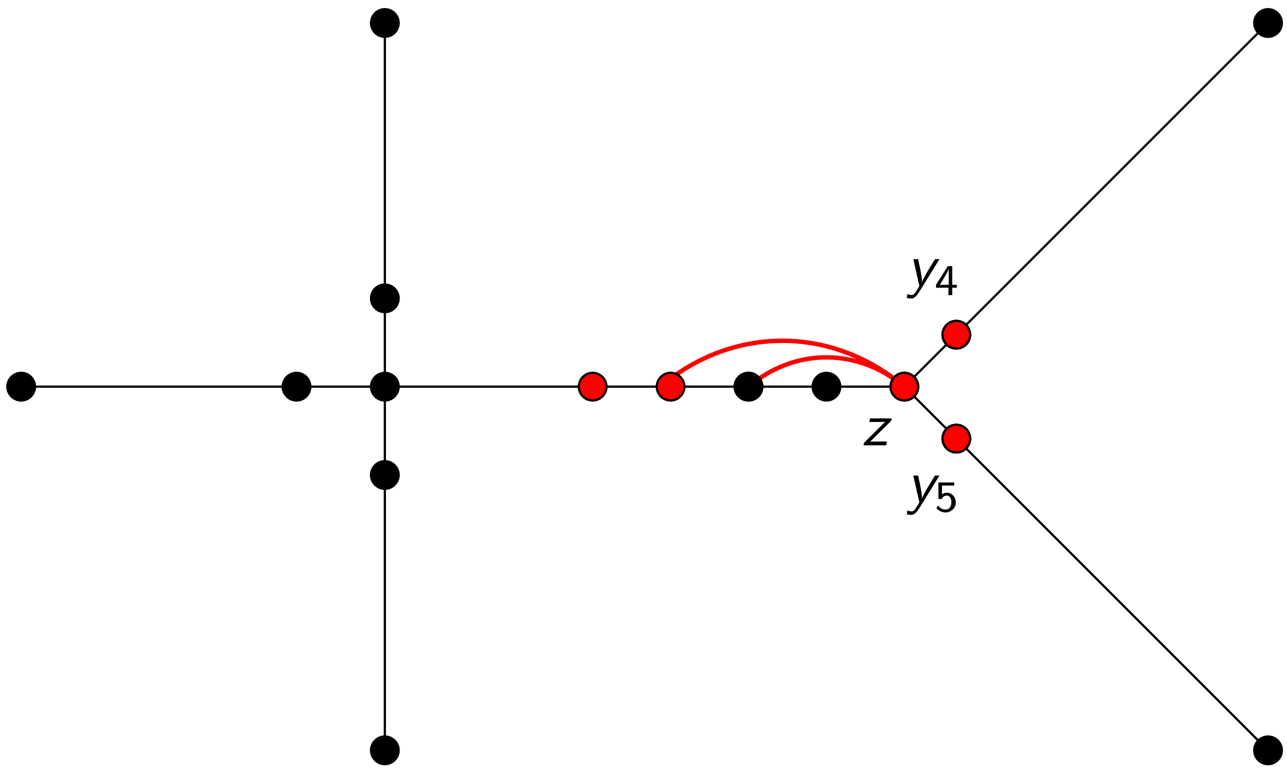
- y_4y_5, y_4v and y_5v, y_4v and y_5v , zv .



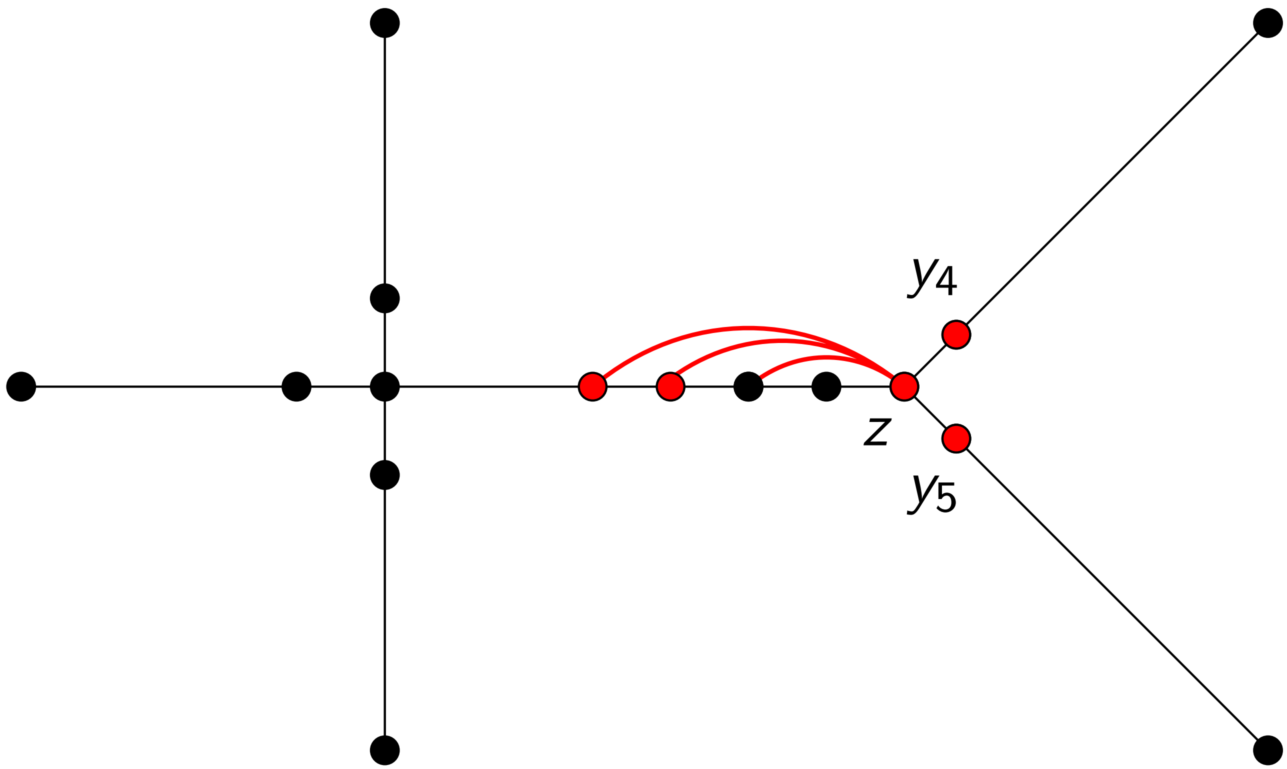
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



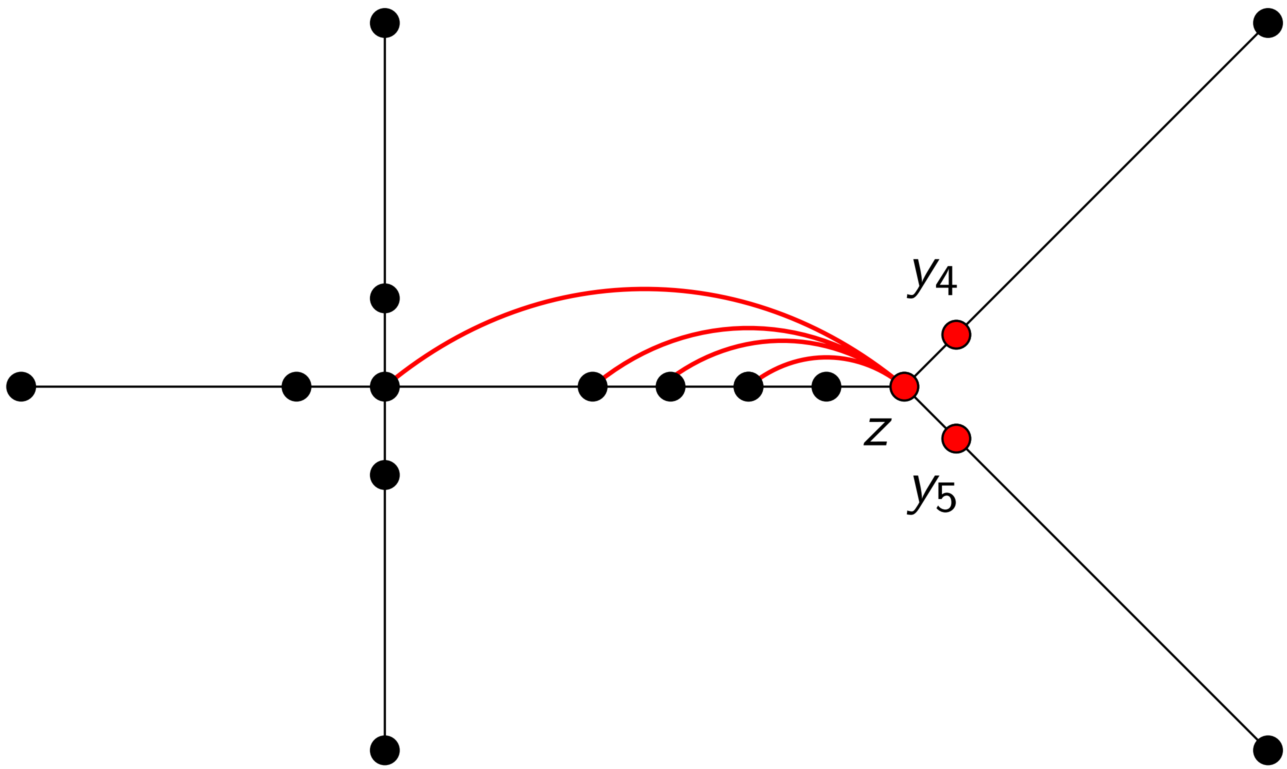
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



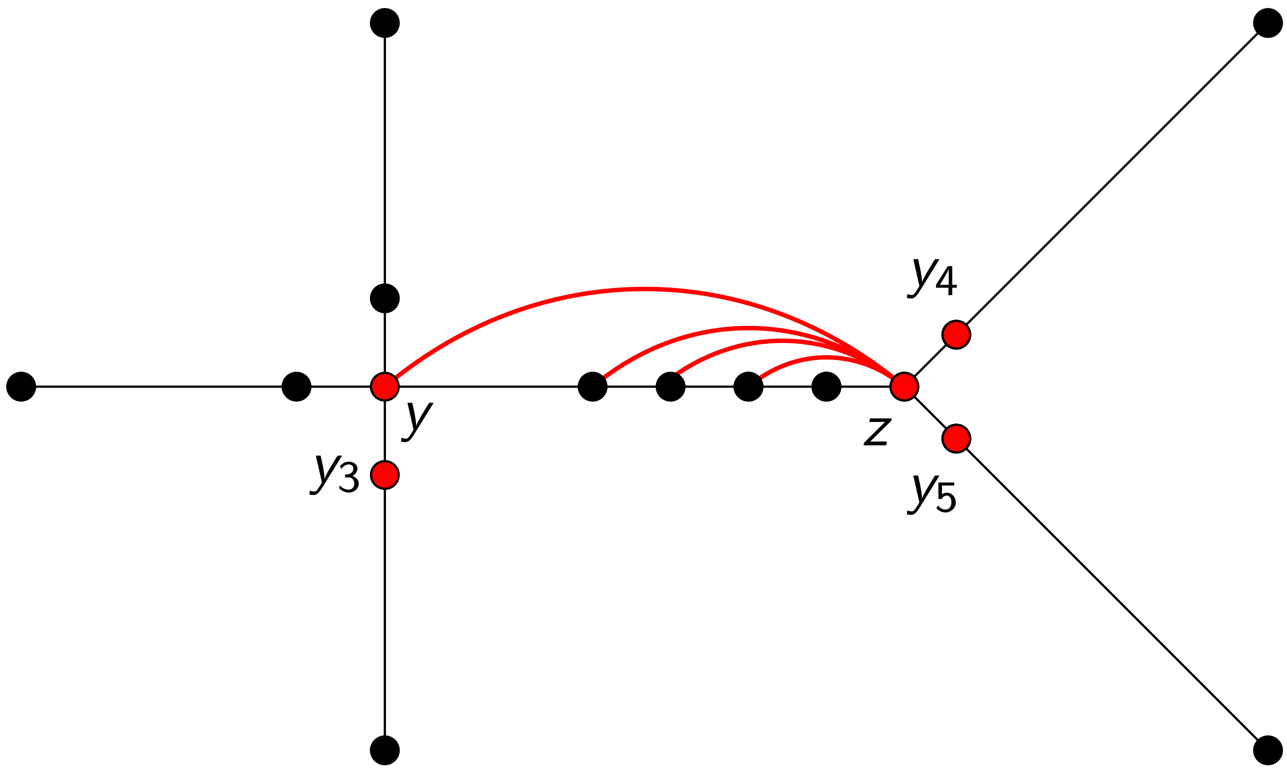
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

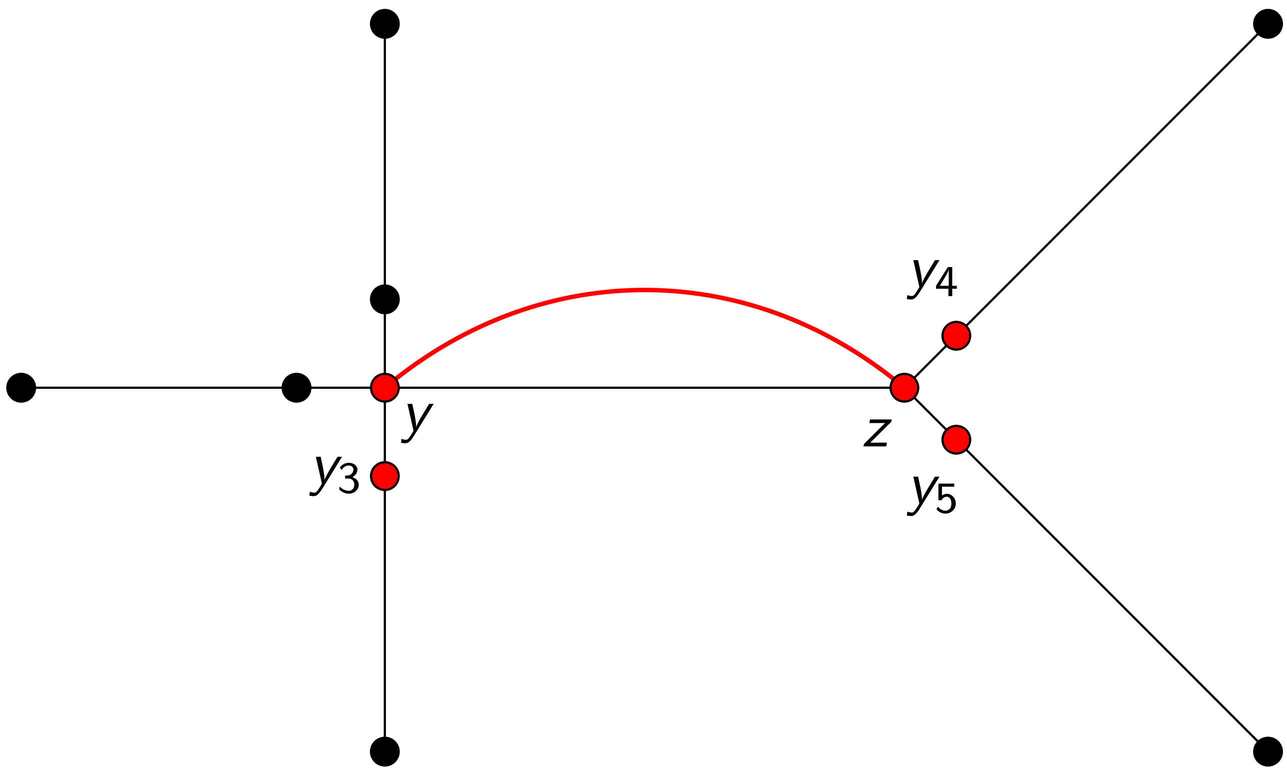


Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.



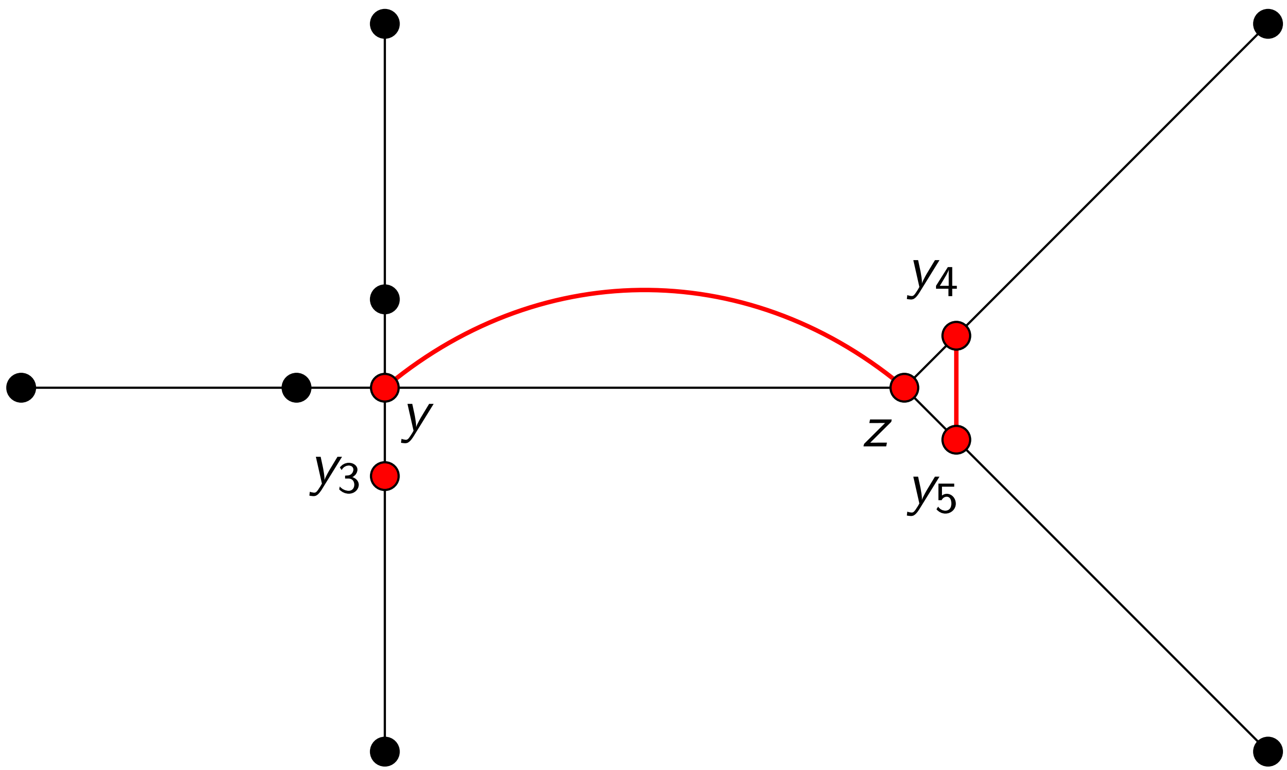
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- y_4y_5, y_4y and y_5y, y_4y_3 and y_5y_3, zy_3



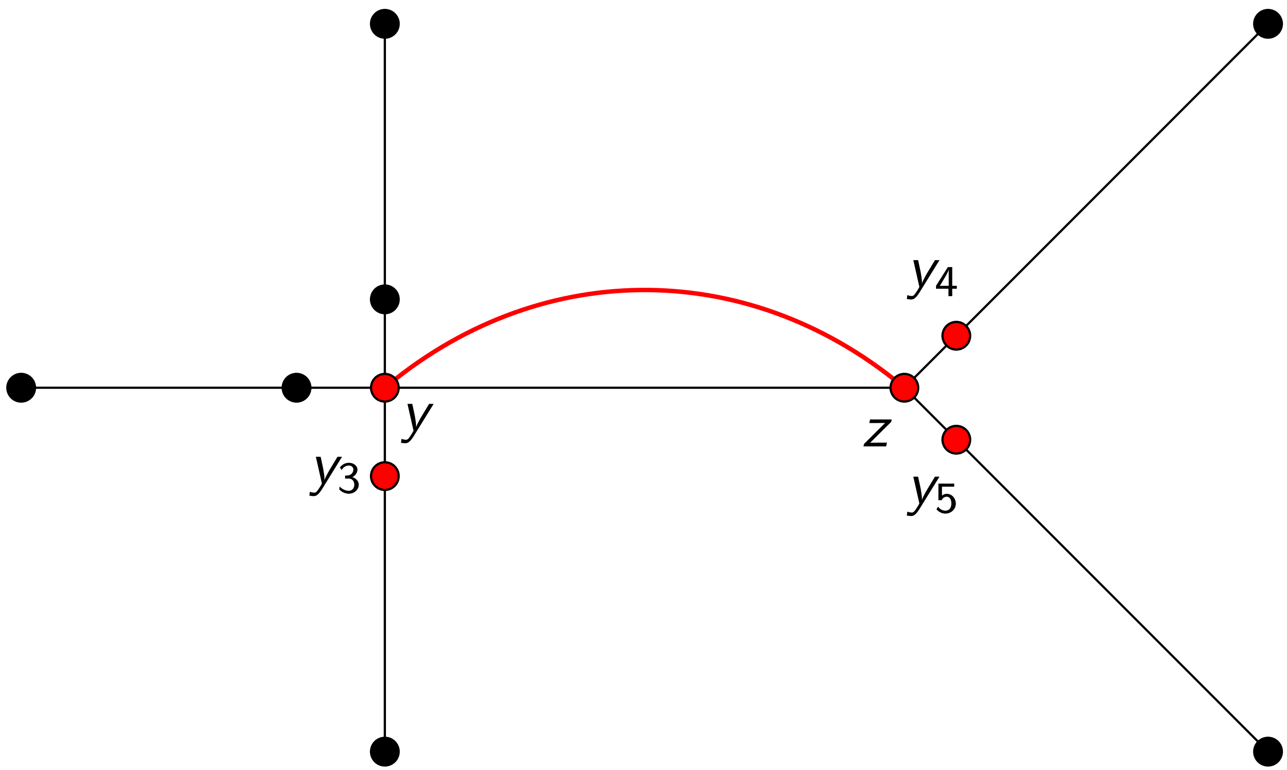
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- y_4y_5, y_4y and y_5y, y_4y_3 and y_5y_3, zy_3



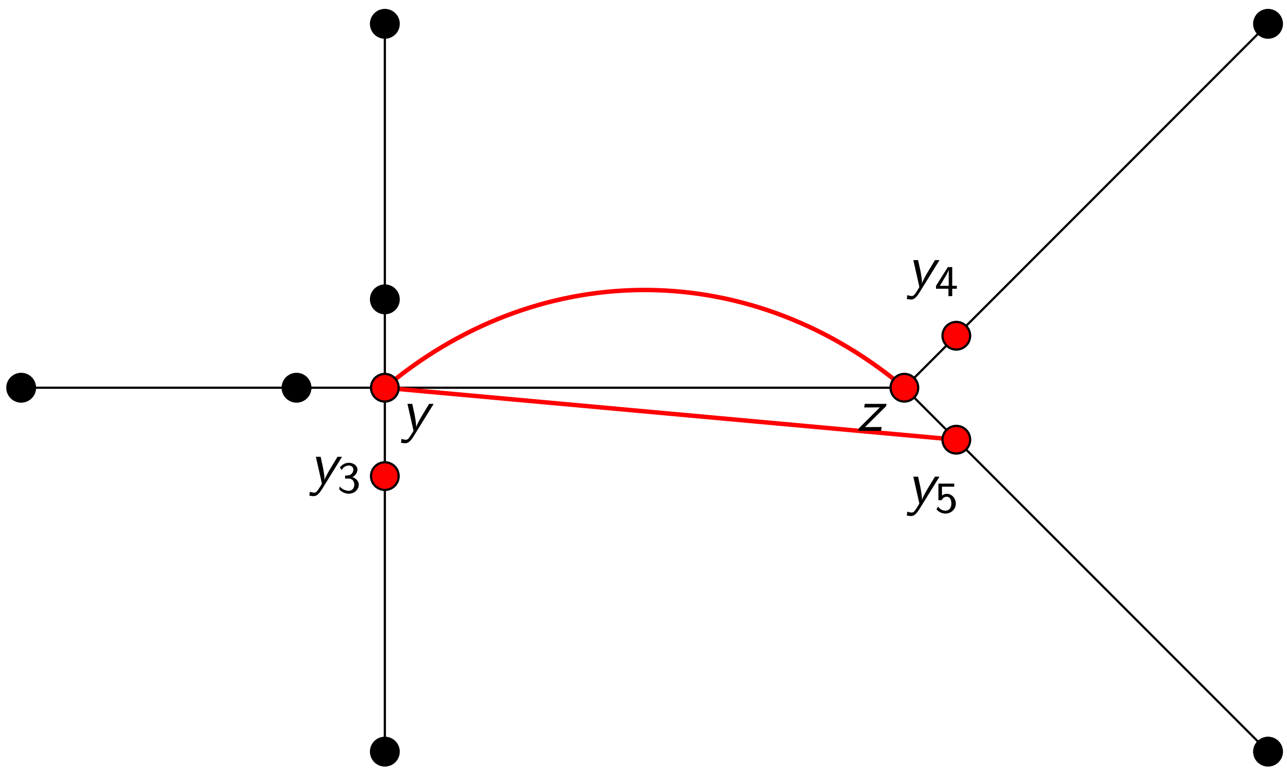
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, $y_4 y$ and $y_5 y$, $y_4 y_3$ and $y_5 y_3$, $z y_3$



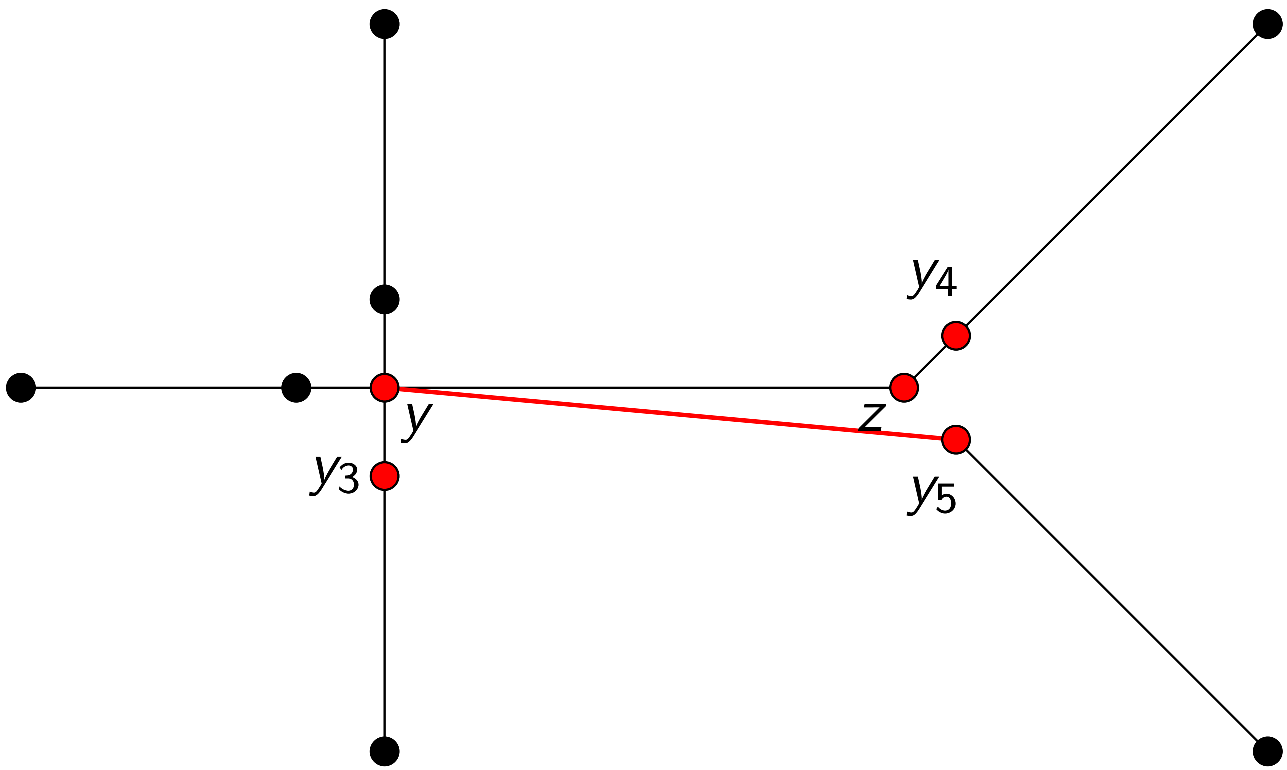
Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, $y_4 y$ and $y_5 y$, $y_4 y_3$ and $y_5 y_3$, $z y_3$



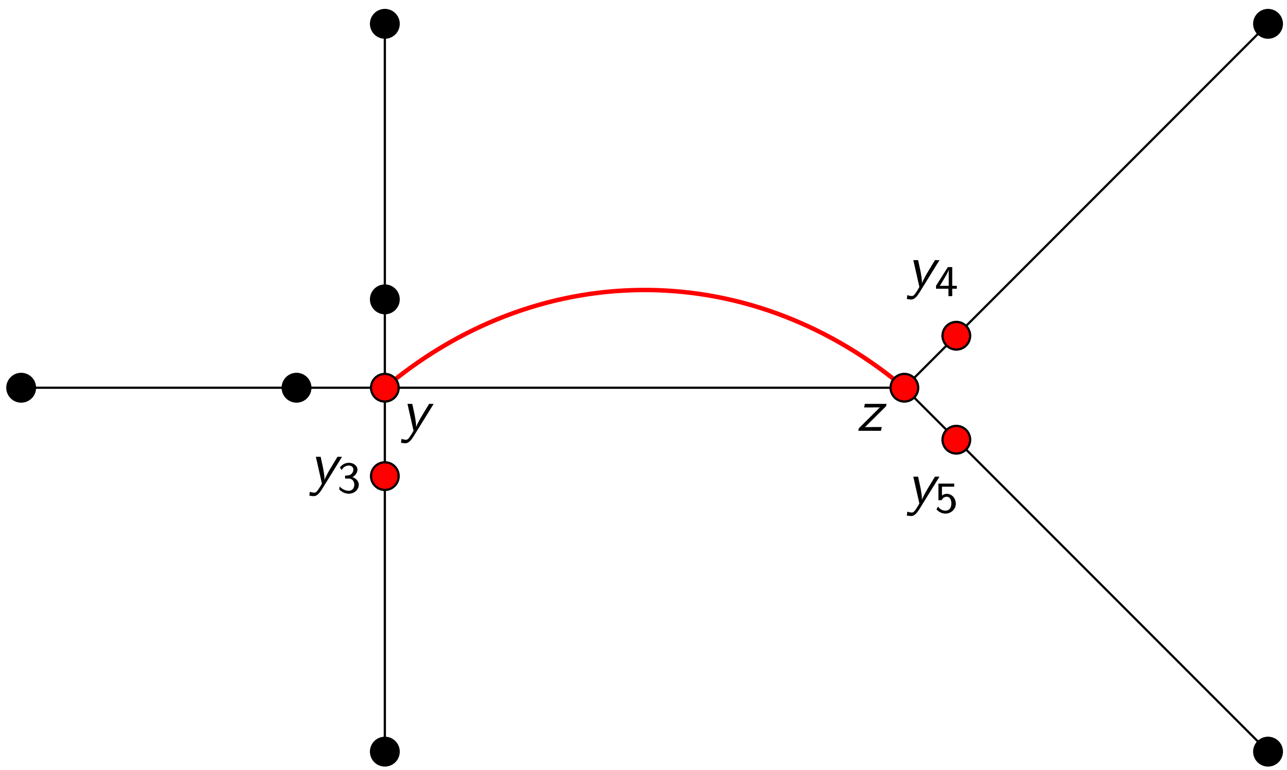
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, y_4y and y_5y, y_4y_3 and y_5y_3, zy_3



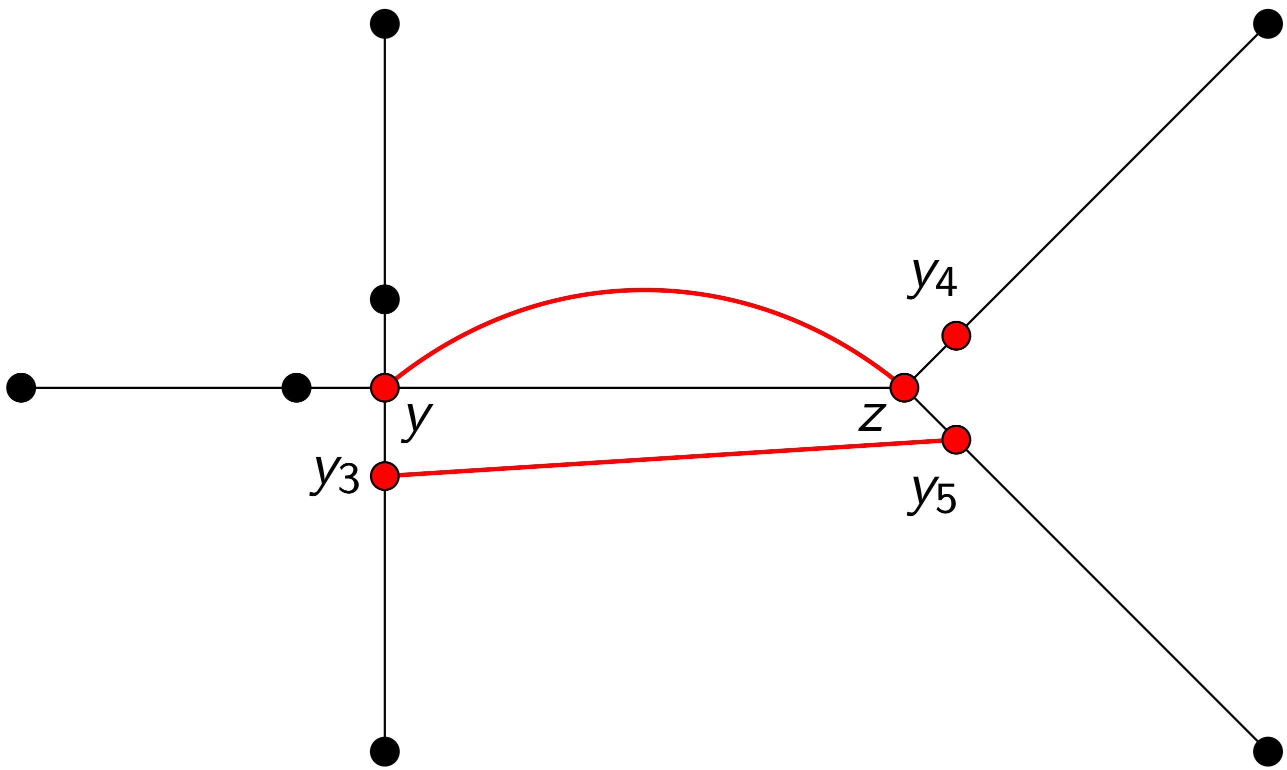
Assume $|E(P)| \quad 2$ and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, ~~$y_4 y$ and $y_5 y$~~, $y_4 y_3$ and $y_5 y_3$, $z y_3$



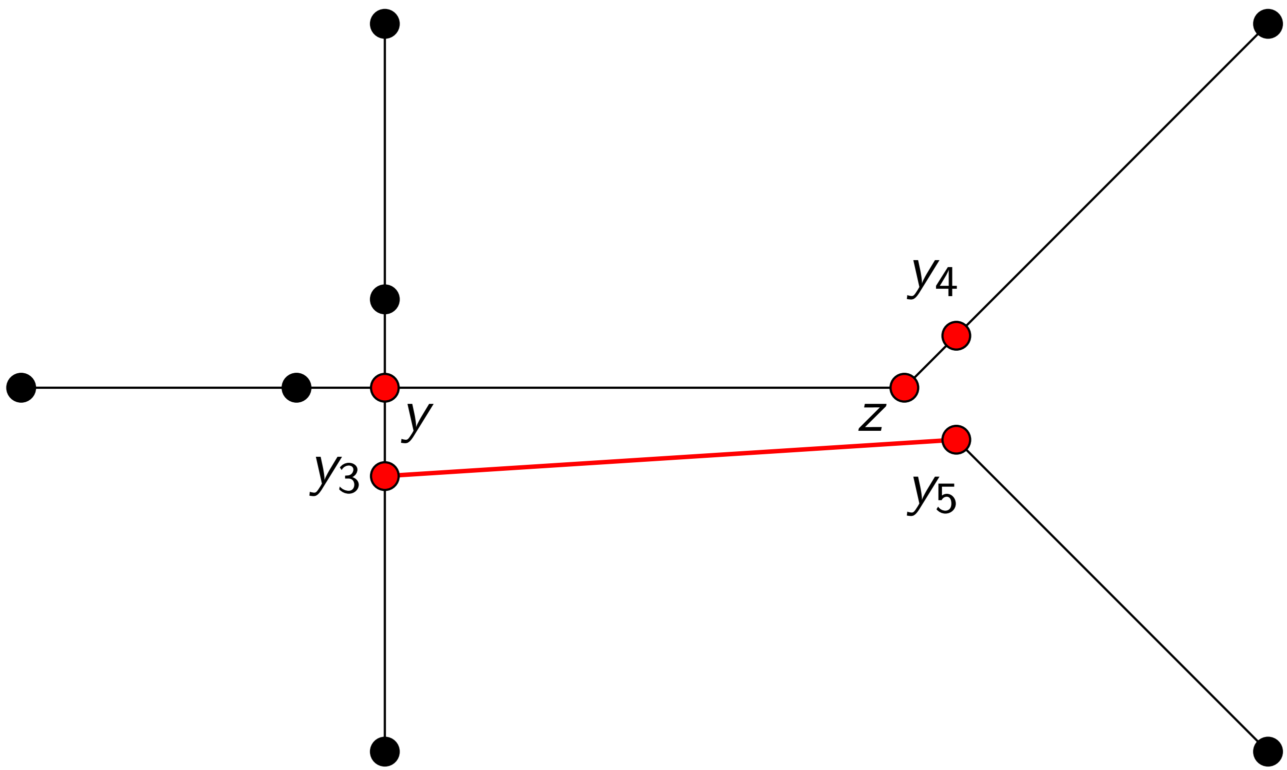
Assume $|E(P)| \quad 2$ and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, ~~$y_4 y$ and $y_5 y$~~, $y_4 y_3$ and $y_5 y_3$, $z y_3$



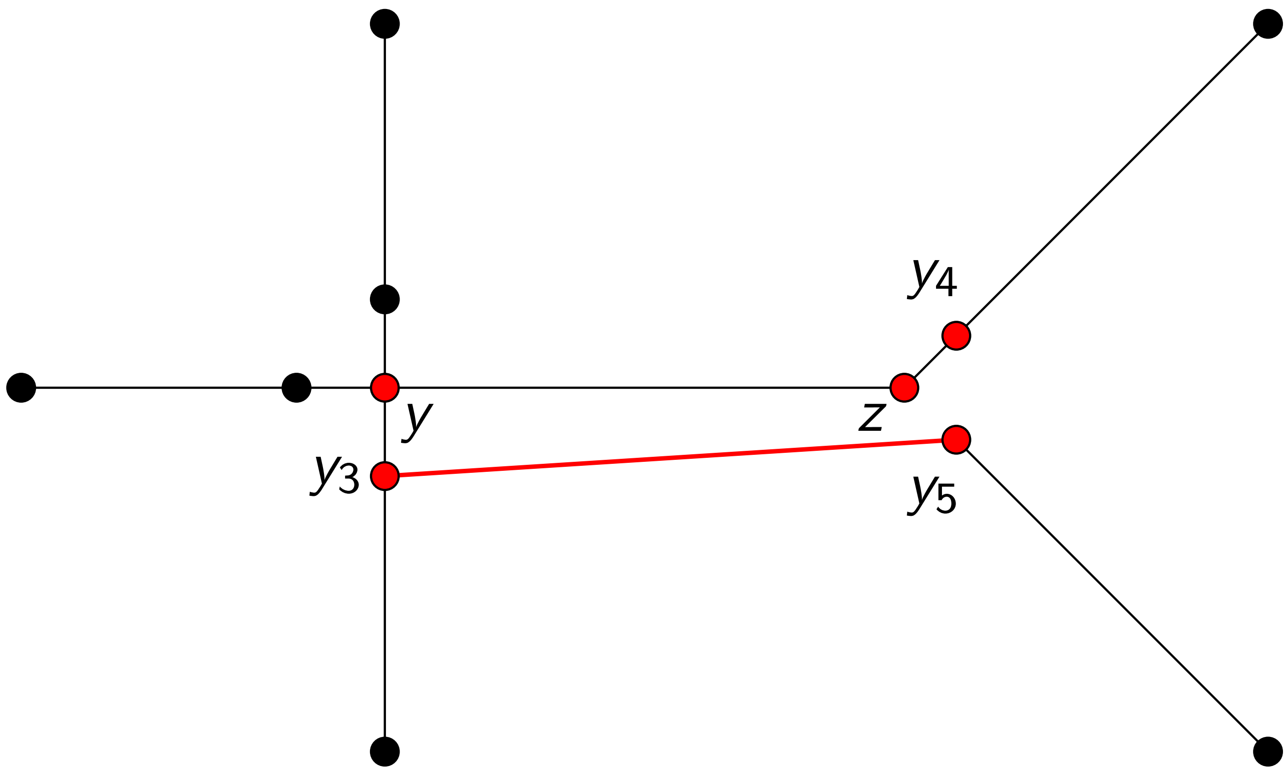
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

- ~~y_4y_5, y_4y and y_5y~~, y_4y_3 and y_5y_3, zy_3



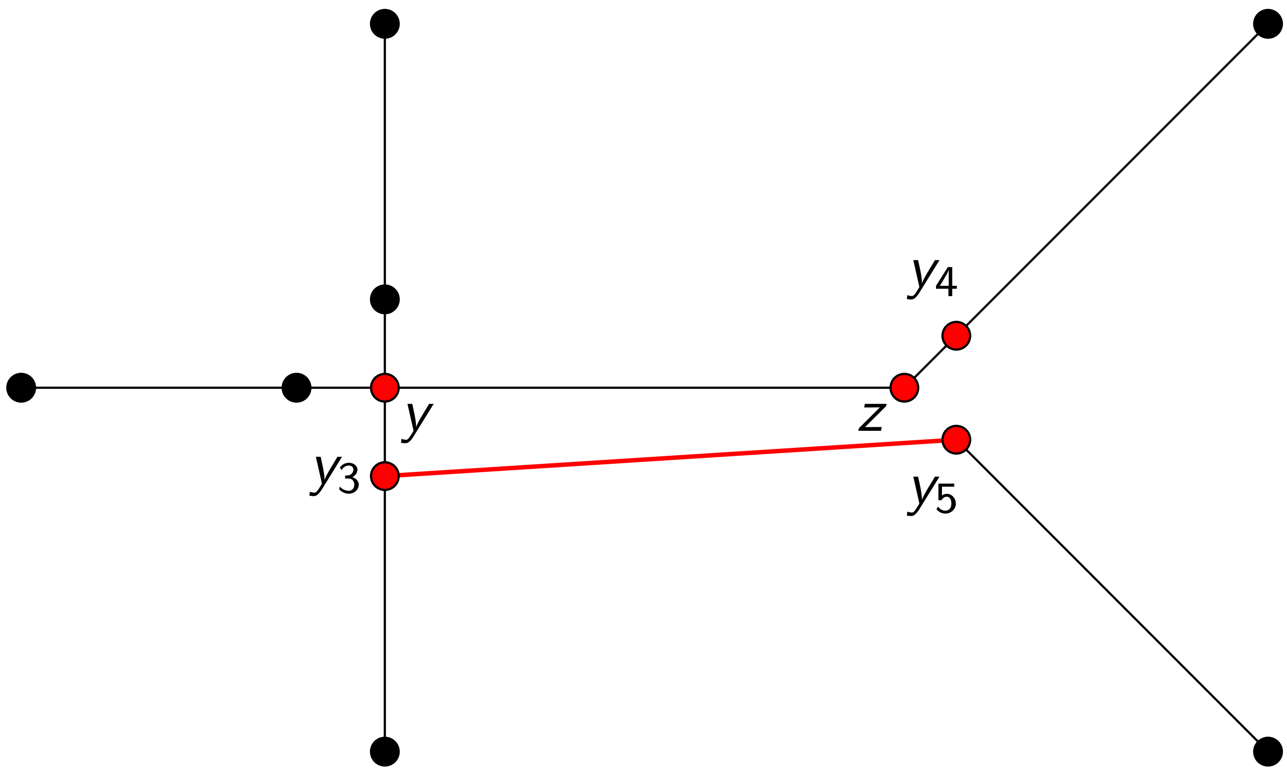
Assume $|E(P)|\quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4y and y_5y~~, y_4y_3 and y_5y_3, zy_3



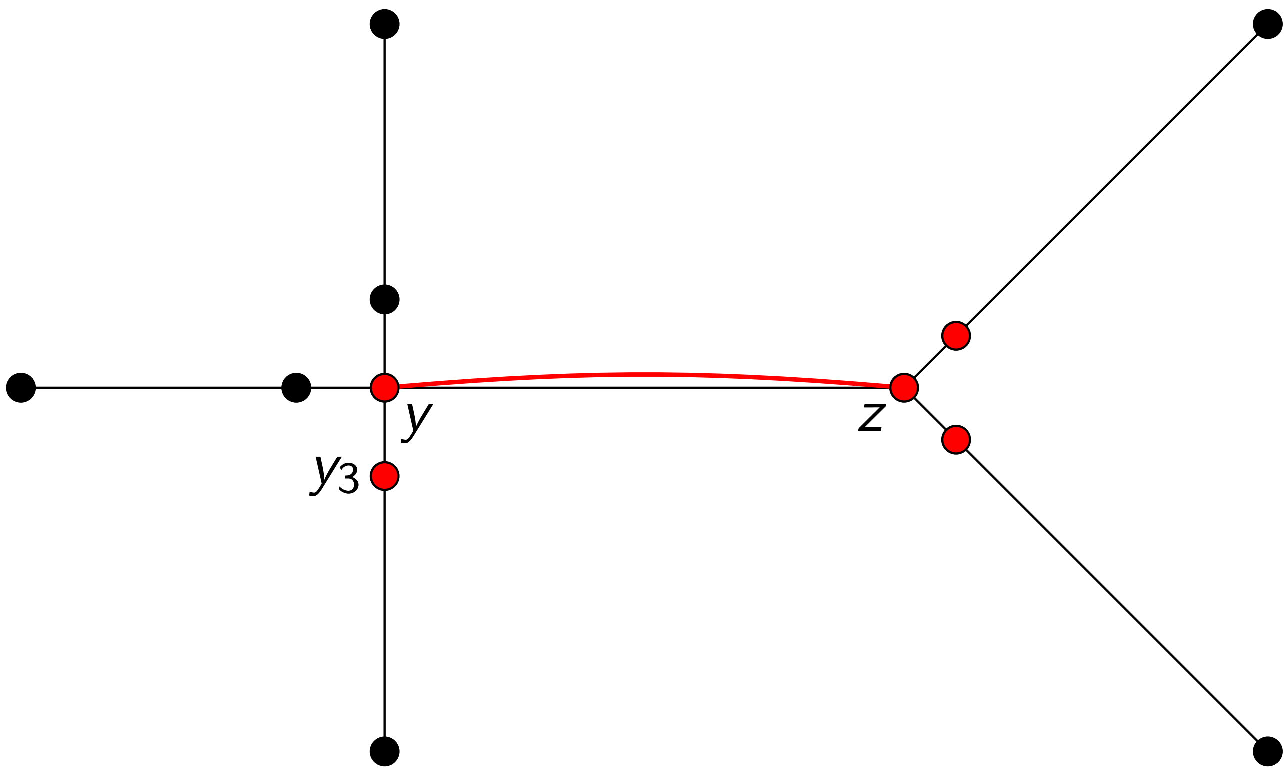
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$. Shorter central path

- ~~y_4y_5~~, ~~y_4y and y_5y~~, y_4y_3 and y_5y_3, zy_3



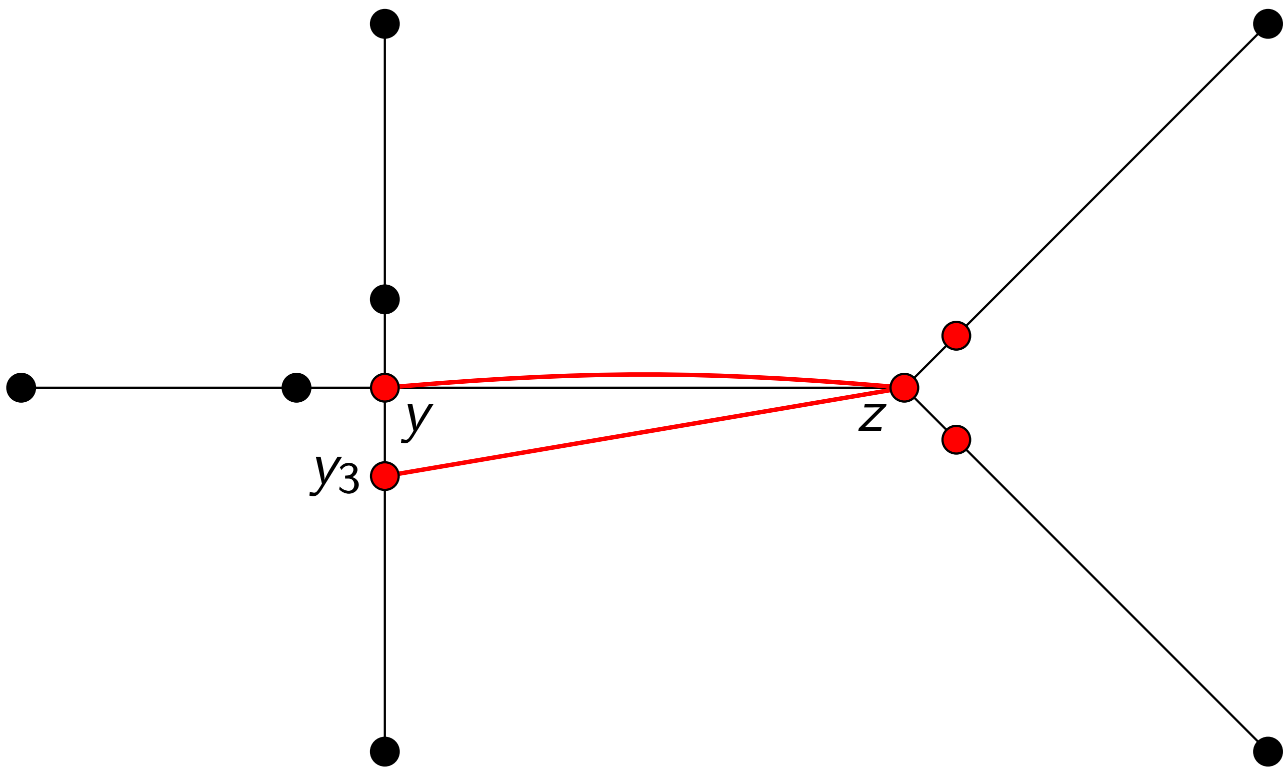
Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4y and y_5y~~, ~~y_4y_3 and y_5y_3~~, zy_3



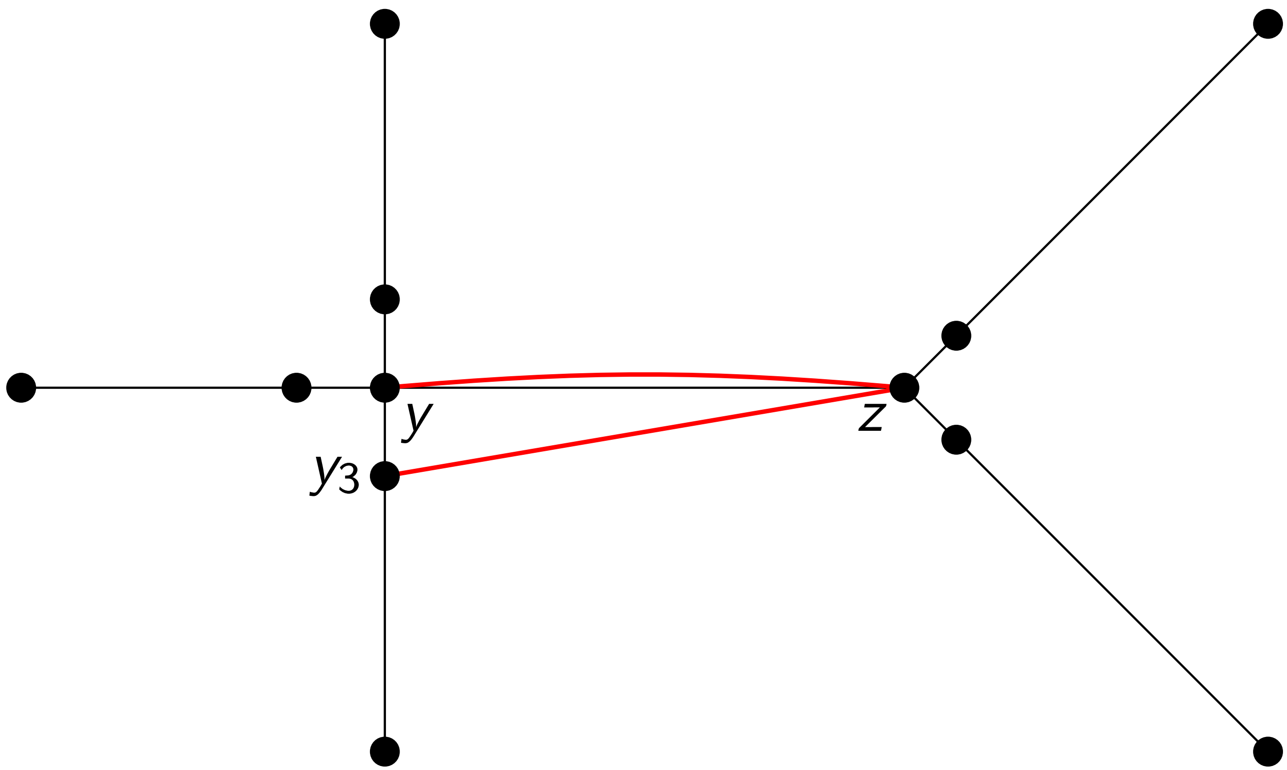
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

- $\cancel{y_4y_5}$, ~~y_4y and y_5y~~, ~~y_4y_3 and y_5y_3~~, zy_3

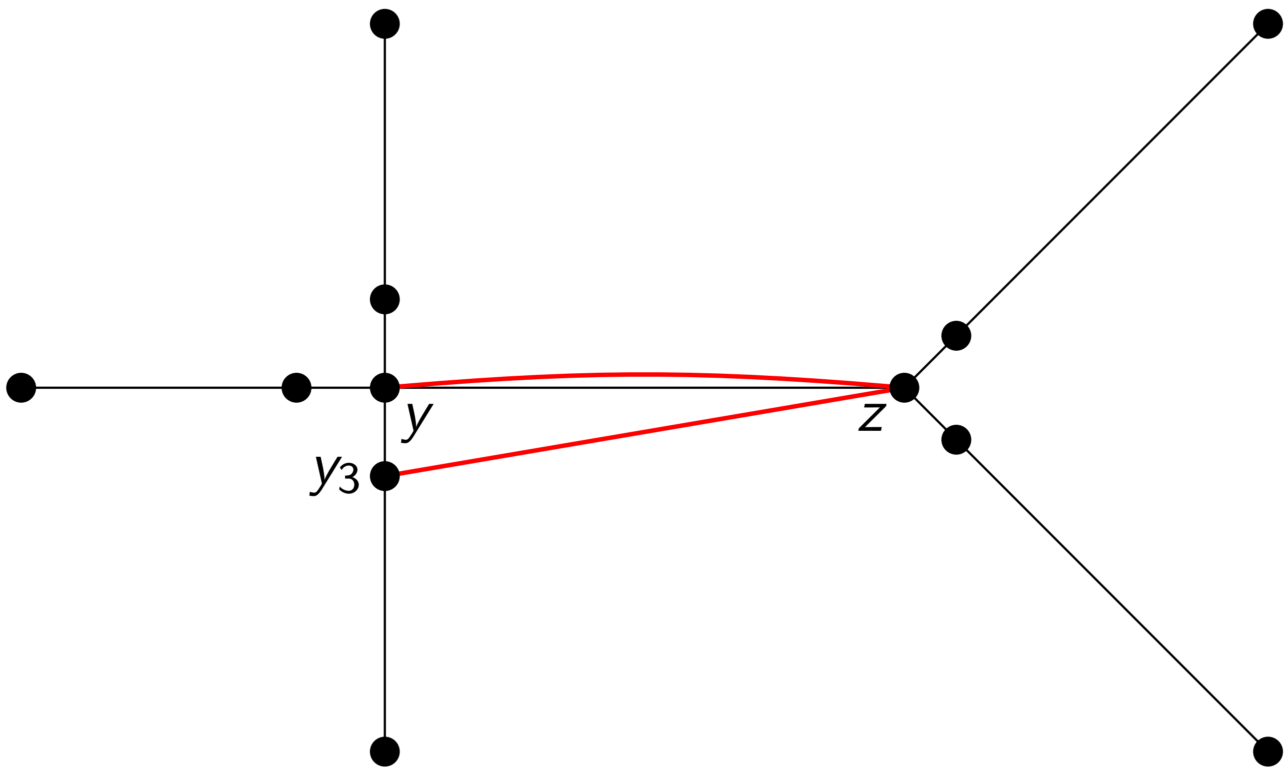


Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.

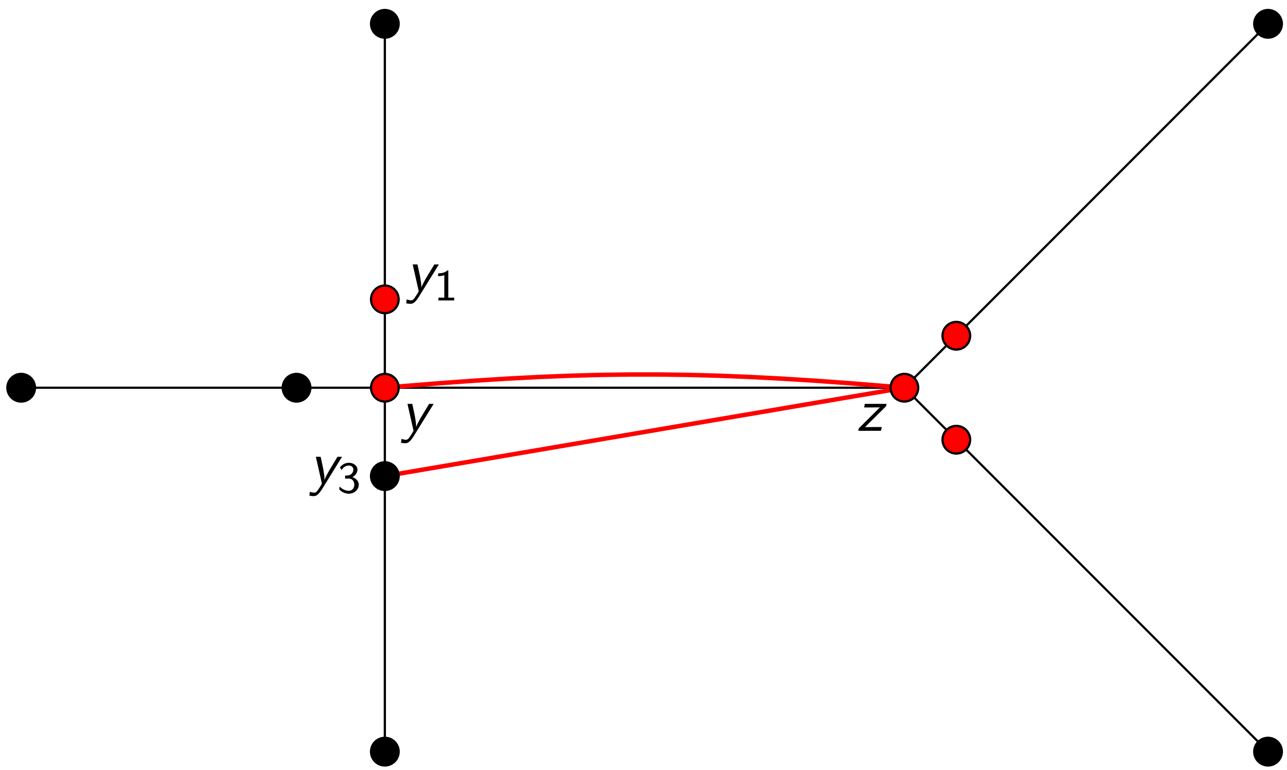
- ~~y_4y_5~~, ~~y_4y and y_5y~~, ~~y_4y_3 and y_5y_3~~, zy_3



Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

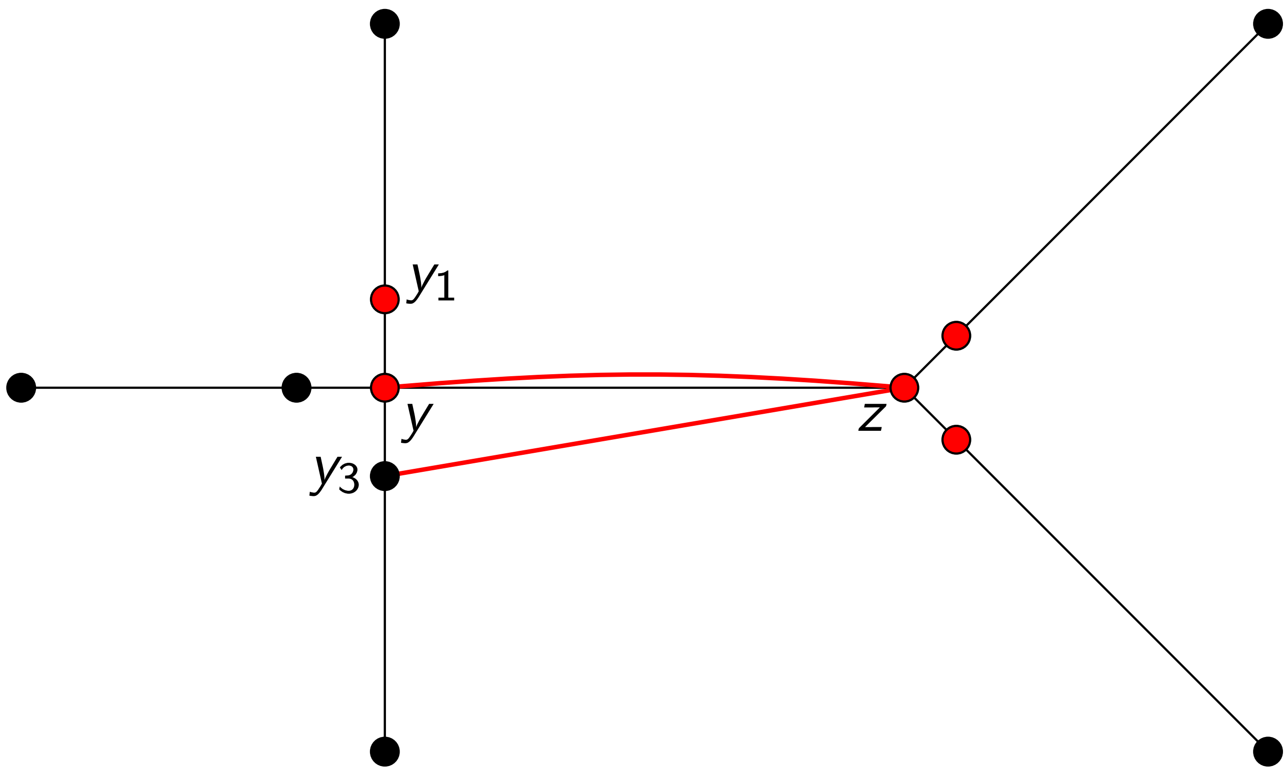


Assume $|E(P)| \quad 2$ and $y_4y_5 \notin E(G)$.



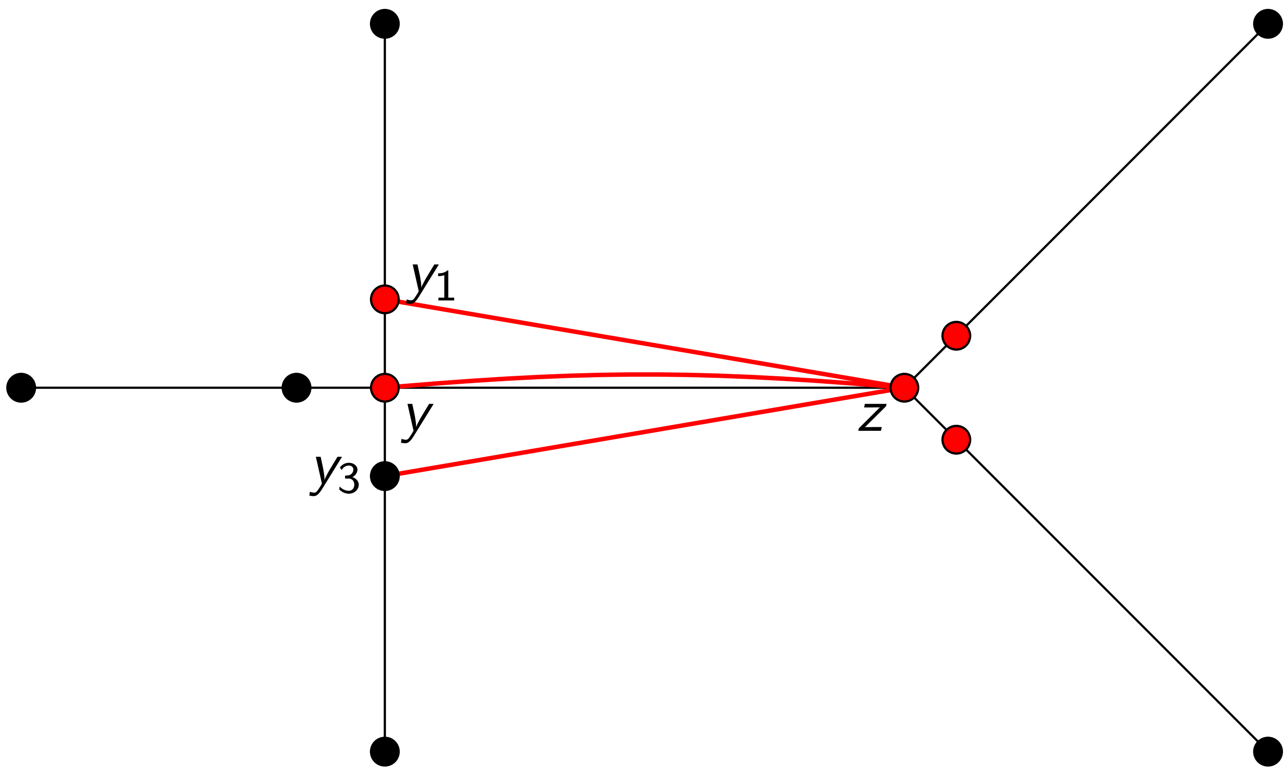
Assume $|E(P)|$ 2 and $y_4y_5 \notin E(G)$.

- ~~y_4y_5~~, ~~y_4y and y_5y~~, ~~y_4y_1 and y_5y_1~~, zy_1



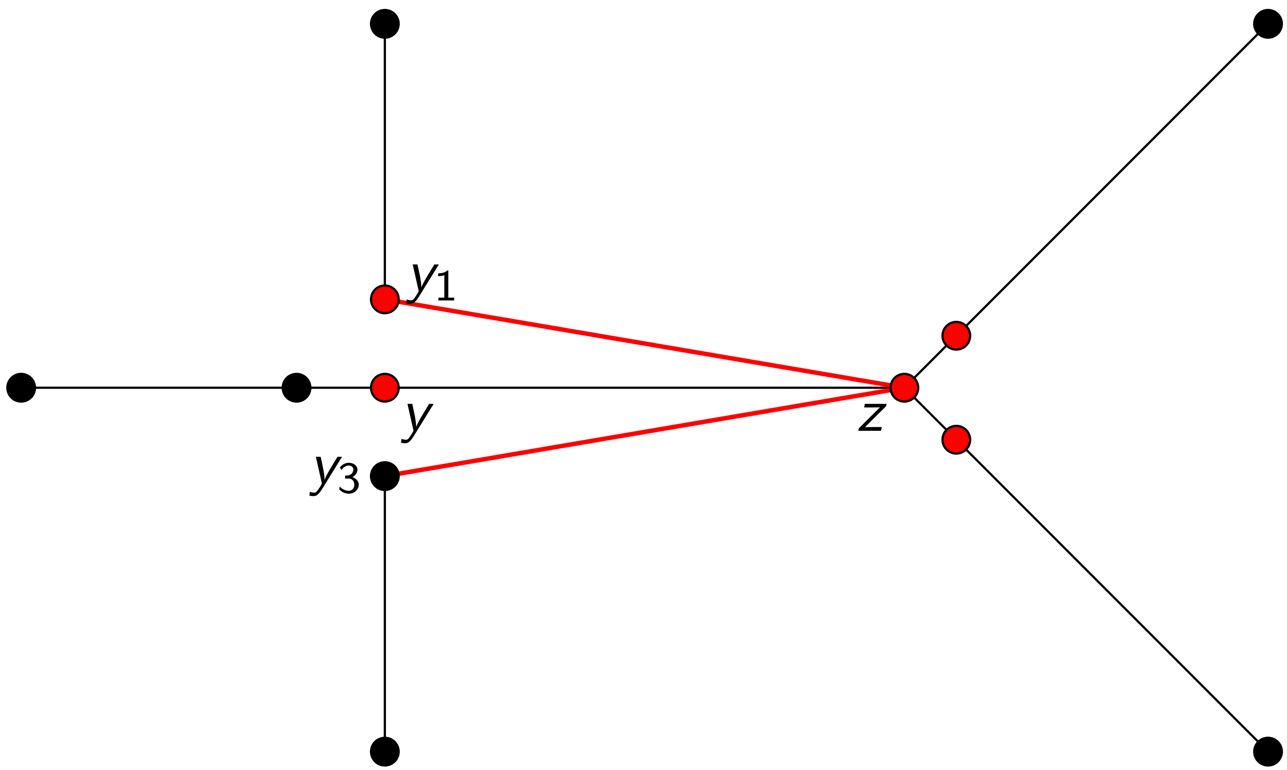
Assume $|E(P)| \quad 2$ and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, ~~$y_4 y$ and $y_5 y$~~, ~~$y_4 y_1$ and $y_5 y_1$~~, zy_1



Assume $|E(P)|$ 2 and $y_4 y_5 \notin E(G)$.

- ~~$y_4 y_5$~~, ~~$y_4 y$ and $y_5 y$~~, ~~$y_4 y_1$ and $y_5 y_1$~~, zy_1



Thank you!! Questions?