Mastermind

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Talking Points

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5. Further research

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Once the codebreaker makes a guess the codemaker will place up to four white or black pegs to the side. (A white peg represents a correct color in the wrong position, and a black peg represents a correct color in the correct position.)

Note: the positions of the white and black pegs don't correlate to the positions of the colored pegs.

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So we will get a black peg whenever $x_j = y_j$ for some position j where $1 \leq j \leq 4$ and a white peg will appear whenever $x_j \neq y_j$ but $x_j = y_k$ for some position k when y_k hasn't been used in another hit.

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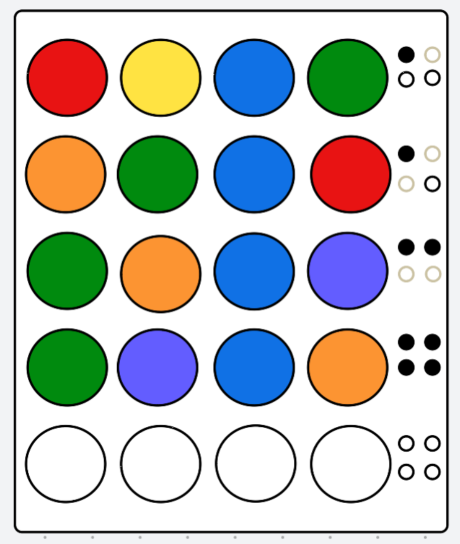
There are $6^4 = 1296$ possible codes using four pegs with six different color choices.

Also note it is impossible to have three black pegs and one white peg, as that peg would have to be in the correct position

Example of a Game of Mastermind



Knuth's algorithm

Donald Knuth created this algorithm in 1976-1977. It is mainly designed for computer use, so it is not an easy algorithm to remember. Also, this is not the optimal strategy for mastermind in the sense that it will not produce the right answer in the shortest number of guesses. This algorithm produces a correct answer in an average of 4.478 moves.

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6. The next guess comes from using the minimax technique.

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	1111	1112	1122	1123	1234
[0,0]	625	256	256	81	16
[0,1]	0	308	256	276	152
[0,2]	0	61	96	222	312
[0,3]	0	0	16	44	136
[0,4]	0	0	1	2	9
[1,0]	500	317	256	182	108
[1,1]	0	156	208	230	252
[1,2]	0	27	36	84	132
[1,3]	0	0	0	4	8
[2,0]	150	123	114	105	96
[2,1]	0	24	32	40	48
[2,2]	0	3	4	5	6
[3,0]	20	20	20	20	20
[4,0]	1	1	1	1	1

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We start with the example 1122 like normal.

$1122 \implies \alpha_{10}$

$1344 \implies \alpha_{01}$

$3526 \implies \alpha_{12}$

$1462 \implies \alpha_{11}$

Which leads us with only one guess left 3632

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7. Return to step 3 until the code is found.

Figure 1

Figure 1

1296(1122: 1,16(1213: 0,0,0,0,0;1,4(1415),3(1145),0;1,3(4115),3(1145);0,1;0),96A,256B,256C;0,36D,208E,256F;
4(1213),32G,114H;0,20I;1) ,

where

A = (2344: 0,2,16(3215: 0,0,0,0,0;1,2,1,1;2,3(3231),2;0,3(3213);1),
14(5215: 0,0,0,0,0;0,1,3(3511),3(3611);1,1,2;0,2;1),4(1515);0,6(2413),
18(2415: 1,1,0,0,0;1,2,3(2253),3(2236);1,2,2;0,1;1),15(2256x);0,4(2234),14(3315x);0,3(2314);0)

B = (2344: 0,7(2335),41(3235: 0,0,2,3(4613),2;0,3(5263),6(3413),6(3416);2,4(3256),6(1336);0,6(1536);1),
44(3516: 1,4(4651),6(6255),1,0;3(5613),7(1461),5(4551),1;3(1113),5(3551),3(4515);0,4(1145);1),
16(5515: 0,0,1,1,0;0,2,2,1;1,1,3(1516);0,3(1516);1);
2,21(3245: 1,3(2436),0,0,0;2,2,2,0;2,3(3234),2;0,3(3243);1),
42(4514: 1,1,7(2456),4(2635),3(2636);0,4(1356),5(4361),6(1635);2,2,3(3614);0,3(4414);1),
34(3315: 0,0,3(5641),4(2566),1;1,4(5361),4(5614),5(6614);2,4(3331),1;0,4(3316);1);
3(2434),13(2425x),23(1545: 0,1,3(2654),3(2353),4(1136);0,2,4(2564),3(2335);0,0,2;0,1;0);
0,9(1335x);1)

C = (3345: 2,20(4653: 2,2,0,0,0;3(4536),3(4534),1,0;2,2,1;0,3(4453);1),
42(6634: 0,3(4566),4(4556),1,0;2,5(4656),6(5653),4(1444);2,5(5636),5(4654);0,4(1413);1),
16(6646: 0,0,1,0,0;0,3(1416),1,1;3(1416),3(5666),2;0,2;0),1;
4(3453),40(3454: 1,5(4535),6(1436),0,0;2,5(4356),6(3536),0;1,3(3564),6(3463);0,4(3456);1),
46(3636: 1,1,3(4364),6(4565),6(4544);0,5(4366),6(1565),6(4546);2,4(3466),3(3556);0,2;1),
18(3656: 0,1,1,1,1;0,3(5665),3(6446),3(4446);0,1,3(4646);0,1;0);
5(3435x),20(3443: 0,0,4(4355),0,0;0,3(3334),4(3356),0;1,2,4(3455);0,1;1),
29(3636: 0,1,3(5365),4(6445),4(1444);0,2,3(3565),4(4645);1,1,4(3446);0,2;0);0,12(3446x);1)

D = (1213: 1,4(1145),3(1415),0,0;0,6(1114x),7(2412x),0;2,4(1145),4(1145x);0,4(1114x);1)

E = (1134: 0,4(1312),24(3521: 1,2,4(4612),0,0;0,3(3312),3(2423),0;2,2,3(4621);0,3(3321);1),
38(2352: 2,4(3226),4(5621),1,0;1,5(2223),7(6242),1;2,4(2323),4(2462);0,2;1),
20(2525: 1,2,1,0,0;0,3(2252),3(2262),0;2,2,2;0,3(2225);1);
4(1341),34(1315: 1,3(4151),4(4161),0,0;1,6(6451),6(1461),0;3(1351),3(1361),2;0,4(1113);1),
32(1516: 2,2,3(2145),0,4(2324);2,4(1661),4(1245),0;3(1561),3(1551),1;0,3(1511);1),
22(1256: 1,0,4(2524),2,0;0,2,4(5224),4(2224);2,0,0;0,2;1);4(1314),12(1315x),12(1235x);0,2;0)

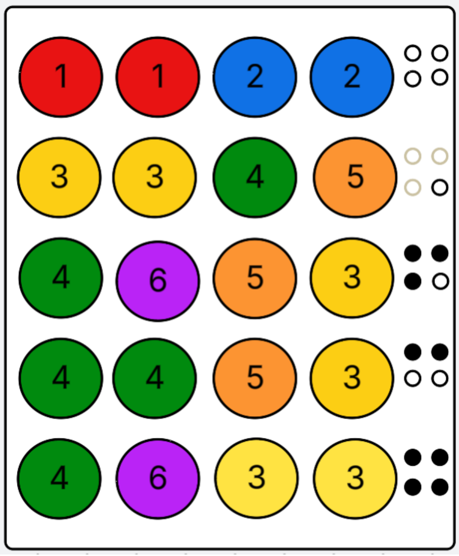
F = (1344: 0,7(1335),41(3135: 0,0,2,3(4623),2;0,3(5163),6(3423),6(3426);2,4(3156),6(1436);0,6(1536);1),
44(3526: 1,4(4652),6(6155),1,0;3(5623),7(1462),5(4552),1;3(1123),5(3552),3(4525);0,4(1145);1),
16(5525: 0,0,1,1,0;0,2,2,1;1,1,3(1516);0,3(1516);1);
2,21(3145: 1,3(1436),0,0,0;2,2,2,0;2,3(3134),2;0,3(3143);1),
42(4524: 1,1,7(1456),4(1635),3(1636);0,4(1356),5(4362),6(1336);2,2,3(3624);0,3(4424);1),
34(3325: 0,0,3(5642),4(1566),1;1,4(5362),4(5624),5(6624);2,4(3332),1;0,4(3326);1);
3(1434),13(1415x),23(1415: 0,0,2,4(3324),0;0,4(1546),4(1356),4(1136);0,2,3(1136);0,0;0);0,9(1335x);1)

G = (1223: 1,4(2145),3(4115),0,0;0,5(2145),6(4512),0;2,4(1245),3(1415);0,3(1145);1)

H = (1234: 2,16(1325: 1,3(4152),3(4162),0,0;1,3(3126),2,0;1,1,1;0,0;0),
20(1325: 0,3(5162),1,0,0;0,2,4(4522),4(4622);0,3(5125),3(2116);0,0;0),
6(2515),0;4(1323),21(1352: 0,1,2,0,0;2,4(1623),2,0;1,3(1323),3(1462);0,2;1),
16(2156x),12(1315x);2,6(3526),8(1536x);0,1;0)

I = (1223: 0,0,0,0,0;1,5(1145x),4(1114x),0;1,3(1415),4(1114x);0,2;0)

Example using Knuth's algorithm



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In 1994 Lai and Koyama developed a slight variation of Knuth's algorithm to bring the average game length down to 4.340 as opposed to Knuth's 4.478 length.

There is really only one change they make to their algorithm, and it is in how they decide which code to choose next. Instead of using the minimax technique like before, they use the code with the smallest *expected* value instead of maximum value.

Further Research

Other research that has been done surrounding this topic:

1. Finding an optimal solution to MM(4, 7) by Geoffroy Ville in 2013

Further research in the topic includes:

1. Finding a generalized formula (to see what happens with more colors or more holes)
2. Research in the Genetic Algorithm for developing a more optimal strategy

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Thank You!

Questions?