Sequences of Sum and Difference Dominated Sets

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We say A is sum-dominated or a **More Sums than Differences** (MSTD) set if $\ A + A \ \ > \ \ A - A\ $.

Example: The Conway Set

Let $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$.

- $A + A \ = \ [0, 28] \setminus \{1, 20, 27\}, \qquad |A + A| \ = \ 26$
- $A - A \ = \ [-14, 14] \setminus \{\pm 6, \pm 13\}, \qquad |A - A| \ = \ 25$

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This set A is called the **Conway set**, and it is the smallest MSTD set in terms of both cardinality and diameter.

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- If there exists an $a^*\in\mathbb{Z}$ such that $A=\{a^*\}-A$, then A is *symmetric* with respect to a^* and A is *sum-difference balanced* ($A+A\ =\ A-A$).

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- If there exists an $a^*\in\mathbb{Z}$ such that $A=\{a^*\}-A$, then A is *symmetric* with respect to a^* and A is *sum-difference balanced* ($A+A \quad = \quad A-A$).
- Several methods for constructing MSTD sets exist.

Nathanson's Construction

Nathanson provides one such construction for MSTD sets.

Theorem (Nathanson, 2006)

Let m, d, k be integers such that $m \geq 4$, $1 \leq d \leq m-1$, $d \neq \frac{m}{2}$, and

$$k \geq 3 \text{ if } d < \frac{m}{2}, \quad k \geq 4 \text{ if } d > \frac{m}{2}.$$

Define

$$\begin{aligned} B &= [0, m-1] \setminus \{d\}, \\ L &= \{m-d, 2m-d, \ldots, km-d\}, \\ a^* &= (k+1)m - 2d, \\ A^* &= B \cup L \cup (a^* - B), \\ A &= A^* \cup \{m\}. \end{aligned}$$

Then A is an MSTD set of integers.

Example

Let $m = 4$, $d = 1$, and $k = 3$. Then:

- $B \;=\; [0, m-1] \setminus \{d\} \;=\; \{\mathbf{0}, \mathbf{2}, \mathbf{3}\}$
- $L \;=\; \{m-d, 2m-d, 3m-d\} \;=\; \{\mathbf{3}, \mathbf{7}, \mathbf{11}\}$
- $a^* \;=\; (k+1)m - 2d \;=\; \mathbf{14}$
- $A^* \;=\; B \cup L \cup (a^* - B) \;=\; \{\mathbf{0}, \mathbf{2}, \mathbf{3}, \mathbf{7}, \mathbf{11}, \mathbf{12}, \mathbf{14}\}$
- $A \;=\; A^* \cup \{m\} \;=\; \{\mathbf{0}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{7}, \mathbf{11}, \mathbf{12}, \mathbf{14}\}$

This is the Conway set!

Problem Statement

At the recent CANT (Combinatorial and Additive Number Theory Conference), Samuel Alexander posed the following: **Find a sequence of sets with $A_{i-1} \subset A_i$ that alternate being sum and difference dominated.**

Filling In

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- We arrive at trivial methods of obtaining the desired sequences by noticing that an interval of integers $[a, b]$ is symmetric with respect to $a + b$.
- Thus, between each step in the sequence we fill in the sets to 'reset' our sum and difference counts.

Filling Method 1

Lemma

Let $[a, b]$ be an interval of integers where $a, b \in \mathbb{Z}$, and $a < b$. Let $p > b + 1$, $p \in \mathbb{Z}$. Then

$$A := [a, b] \cup \{p\}$$

is difference-dominated.

Filling Method 1

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is difference-dominated.

Consider any MSTD set A_1, where $A_1 \subset [0, n]$, $n \in \mathbb{N}$. From the lemma, $A_2 := [0, n] \cup \{p\}$ is difference dominated and contains A_1 if $p > n + 1$.

Filling Method 1

Next, we obtain an MSTD set A_3 from A_2:

- Let $m = p + 2$ if p is odd, $m = p + 5$ if p is even
- Set $2 \leq d \leq m - 3$
- Set $k \geq 2$, and $a^* = (k + 3)m - 2d$.

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We apply Nathanson's Construction for MSTD sets:

- $B = [0, m - 1] \setminus \{d\}$.
- $A_3^* = B \cup \{2m - d, 3m - d, \ldots, (k + 1)m - d\} \cup (a^* - B)$.
- $A_3 = A_3^* \cup \{m\}$.

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- $B = [0, m - 1] \setminus \{d\}$.
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- $A_3 = A_3^* \cup \{m\}$.

Thus, A_3 is MSTD. We have $A_1 \subset A_2 \subset A_3$, and we can extend this sequence infinitely by setting $n = \max(A_3)$ and repeating the steps used to generate A_2 and A_3.

Filling Method 1 Example

- $A_1 = \{0, 2, 3, 4, 7, 11, 12, 14\}$, $p = 17$
- $A_2 = [0, 14] \cup \{17\}$
- $A_3 = [0, 18] \setminus \{16\} \cup \{22, 41\} \cup [45, 63] \setminus \{47\} \cup \{19\}$
- $A_4 = [0, 63] \cup \{65\}$

Filling Method 1 Example Sequence

Table: Filling in Method 1 Example Sequence

Set	$A_i + A_i$	$A_i - A_i$	Cardinality	Diameter	Density
A_1	26	25	8	14	0.571
A_2	33	35	16	17	0.941
A_3	126	125	39	63	0.619
A_4	130	131	65	65	1.000
A_5	414	413	135	207	0.652
A_6	418	419	209	209	1.000
A_7	1278	1277	423	639	0.662
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.667

Sets Not Contained in $[0, n]$

Since $x \cdot A + \{y\}$ is MSTD for any MSTD set A, we can translate *any* MSTD set to fit within $[0, n]$ and use it to start this method.

Filling Method 2

Definition (P_n Set)

A set of integers is called a ***P_n* set** if both its sumset and difference set contain all possible elements except for the first and last n.
Equivalently, for a set A with $a = \min A$ and $b = \max A$,

- $[2a + n,\ 2b - n] \subset A + A$
- $[-(b - a) + n, (b - a) + n] \subset A - A$

For instance, if $A = \{0, 1\}$, then $A + A = \{0, 1, 2\}$ and $A - A = \{-1, 0, 1\}$. As such, A is both a P_0 and a P_1 set.

Filling Method 2

Theorem

Let $A_1 = L \cup R$ *where* $L \subset [1, n]$ *and* $R \subset [n+1, 2n]$*, with* $1, 2n \in A_1$ *and* $n \notin A_1$*. Suppose that* A_1 *is a* P_n *set and MSTD.*

For $l \geq 1$*, define*

$$\begin{aligned} A_{2l} &= ([(1-l)n, (l+1)n] \setminus \{n\}) \cup \{(l+2)n\} \\ A_{2l+1} &= (L - ln - 1) \cup ([(1-l)n, (l+1)n] \setminus \{n\}) \cup (R + ln). \end{aligned}$$

Then the sequence of sets $A_1 \subset A_2 \subset \cdots$ *alternates between being MSTD and MDTS.*

Filling Method 2 Example Sequence

- $A_1 = \{1, 3, 4, 8, 9, 12, 13, 15, 18, 19, 20\}$ is P_{10} and MSTD, and contains 1 and 20 but not 10
- $A_2 = [0, 20] \setminus \{10\} \cup \{30\}$
- $A_3 = \{-10, -8, -7, -3, -2\} \cup [0, 20] \setminus \{10\} \cup \{22, 23, 25, 28, 29, 30\}$

Filling Method 2 Example Sequence

Table: Filling in Method 2 Example Sequence

Set	$A_i + A_i$	$A_i - A_i$	Cardinality	Diameter	Density
A_1	38	37	11	16	0.688
A_2	52	61	21	30	0.700
A_3	80	79	31	40	0.775
A_4	92	101	41	50	0.820
A_5	120	119	51	60	0.850
A_6	132	141	61	70	0.871
A_7	160	159	71	80	0.888
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 1.000

Non-Filling Method 1

We now add the constraint that we are not allowed to fill in sets to obtain the desired sequence.

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Theorem

Let A_1 be MSTD with $0 \in A_1$, and $n > \max(A_1)$ satisfying:

- $|(A_1 + A_1) \mod n| = |(A_1 - A_1) \mod n|$
- $2y - x - 1 > |A_1 + A_1| - |A_1 - A_1|$ *where:*
    - $x = |a \in A_1 : n + a \notin A_1 + A_1|$
    - $y = |b \in A_1 : n - b \notin A_1 - A_1|$
- *Then,* $A_2 = A_1 \cup \{n\}$ *is difference-dominated,* $A_3 = A_1 + \{0, n\}$ *is sum-dominated.*

Non-filling Method 1

- We have $A_1 \subset A_2 \subset A_3$.
- For $l \geq 2$, let

$$
\begin{aligned}
A_{2l} &= A_{2l-1} \cup \{ln\} \\
A_{2l+1} &= A_{2l-1} \cup (A_1 + ln)
\end{aligned}
$$

Non-filling Method 1

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- For $l \geq 2$, let

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- Clearly, $A_{2l} \subset A_{2l+1}$. We proved A_{2l} and A_{2l+1} continue to alternate being sum- and difference-dominated.
- Using these constructions, we are able to generate the desired infinite sequence.

Non-filling Method 1 Example

$A_1 = \{0, 2, 3, 4, 7, 11, 12, 14\}$, $n = 17$.

- $A_2 = \{0, 2, 3, 4, 7, 11, 12, 14, 17\}$
- $A_3 = \{0, 2, 3, 4, 7, 11, 12, 14, 17, 19, 20, 21, 24, 28, 29, 31\}$
- $A_{2l+1} = 17 \cdot [0, l] + \{0, 2, 3, 4, 7, 11, 12, 14, 17\}$
- $A_{2l} = A_{2l-1} \cup \{ln\}$

Non-filling Method 1 Example

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A_1	26	25	8	14	0.571
A_2	30	33	9	17	0.529
A_3	60	59	16	31	0.516
A_4	64	67	17	34	0.500
A_5	94	93	24	48	0.500
A_6	98	101	25	51	0.490
A_7	128	127	32	65	0.492
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.471 $(8/17)$

Non-Filling Method 2

Theorem

Suppose there are sets $L, R \subset [0, n]$ *such that*

- $0, n \in L, R$
- $[0, n-1] \subset (L + L)$
- $[0, n-1] \subset (R + R)$
- $[0, n-1] \not\subset [R + L]$

Non-Filling Method 2

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- $[0, n-1] \subset (L + L)$
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- $[0, n-1] \not\subset [R + L]$

Then for sufficiently large $m \geq n$ *and for all* $k \geq 1$*, set*

$$A_{2k-1} \;=\; L \;\cup\; [n, m] \;\cup\; n \cdot [1, k] + (m - R).$$

Then A_{2k-1} *is MSTD, and there may exist* A_{2k} *which is MDTS such that* $A_{2k-1} \subset A_{2k} \subset A_{2k+1}$.

Non-Filling Method 2

- The sumset of $A_{2k-1} + A_{2k-1}$ is

$$
\begin{aligned}
& [0, 2m+n] \;\cup\; n \cdot [2, 2k] + (m-R) + (m-R) \\
=\; & [0, 2m+n] \;\cup\; n \cdot [2, 2k] + 2m - (R+R) \\
=\; & [0, 2m+n] \;\cup\; [2m+n+1, 2m+2kn] \\
=\; & [0, 2m+2kn] \;=\; [0, 2 \cdot \max A_{2k-1}]
\end{aligned}
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Non-Filling Method 2

- The sumset of $A_{2k-1} + A_{2k-1}$ is

$$\begin{aligned} & [0, 2m+n] \ \cup \ n \cdot [2, 2k] + (m-R) + (m-R) \\ = \ & [0, 2m+n] \ \cup \ n \cdot [2, 2k] + 2m - (R+R) \\ = \ & [0, 2m+n] \ \cup \ [2m+n+1, 2m+2kn] \\ = \ & [0, 2m+2kn] \ = \ [0, 2 \cdot \max A_{2k-1}] \end{aligned}$$

- Since we have all possible sums, it suffices to have one element missing from $A_{2k-1} - A_{2k-1}$.

Non-Filling Method 2

- We know $A_{2k-1} - A_{2k-1} \subseteq [-(m+nk), m+nk]$.
- Elements of $A_{2k-1} - A_{2k-1}$ which are larger than $m+(k-1)n+1$ must belong to $(nk+m-R)-L \ = \ nk+m-(R+L)$.
- Due to $[0, n-1] \not\subset [R+L]$, at least one such element is missing.

Non-Filling Method 2 Example

For $l \geq 1$, let

$$\begin{aligned} A_{2l-1} &= \{0,1,2,5,8,9,10\} \cup (8 \cdot [1,l] + \{6,7,9,10\}), \\ A_{2l} &= A_{2l-1} \cup \{8l+14\} \end{aligned}$$

This corresponds to $n = 8$, $L = \{0,1,2,5,8\}$, $R = \{0,1,3,4,8\}$, and $m = 10$.

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This corresponds to $n = 8$, $L = \{0,1,2,5,8\}$, $R = \{0,1,3,4,8\}$, and $m = 10$.

- $A_1 = \{0,1,2,5,8,9,10,14,15,17,18\}$
- $A_2 = \{0,1,2,5,8,9,10,14,15,17,18,22\}$
- $A_3 = \{0,1,2,5,8,9,10,14,15,17,18,22,23,25,26\}$

Non-Filling Method 2 Example

Table: Non-Filling in Method 2 Example Sequence

Set	$A_i + A_i$	$A_i - A_i$	Cardinality	Diameter	Density
A_1	36	35	11	18	0.611
A_2	40	41	12	22	0.545
A_3	52	51	15	26	0.577
A_4	56	57	16	30	0.533
A_5	68	67	19	34	0.559
A_6	72	73	20	38	0.526
A_7	84	83	23	42	0.548
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.500

Non-filling Method 3

Theorem

Let A be an MSTD set built via Nathanson's construction, with the additional constraints that $m \equiv 0 \pmod 4$ and $d \in \left\{\frac{m}{4}, \frac{3m}{4}\right\}$. Then

$$A_1 = A \cup \{-d, (k+1)m - d\}$$

is MSTD. For $r \geq 1$, define

$$A_{2r} := A_{2r-1} \cup \{(k+r+1)m - d\},$$
$$A_{2r+1} := A_{2r-1} \cup \{-rm - d, (k+r+1)m - d\}.$$

Then A_{2r} is MDTS, A_{2r+1} is MSTD, and

$$A_1 \subset \cdots \subset A_{2r-1} \subset A_{2r} \subset A_{2r+1} \subset \cdots$$

forms the desired alternating sequence.

Non-filling Method 3 Example

- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.

Non-filling Method 3 Example

- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.
- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.

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- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.
- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.
- $A_1 = A \cup \{-1, 15\}$ is MSTD. $A_2 = A_1 \cup \{19\}$ is difference-dominated. $A_3 = A_2 \cup \{-5\}$ is MSTD.

Non-filling Method 3 Example

- $A = \{0, 2, 3, 4, 7, 11, 12, 14\}$ can be expressed as $\{0, 2, 3\} \cup \{3, 7, 11\} \cup \{11, 12, 14\} \cup \{4\}$.
- By extending the arithmetic progression $\{3, 7, 11\}$, we are able to generate the desired sequence.
- $A_1 = A \cup \{-1, 15\}$ is MSTD. $A_2 = A_1 \cup \{19\}$ is difference-dominated. $A_3 = A_2 \cup \{-5\}$ is MSTD.
- For $l \geq 2$, $A_{2l} = A_{2l-1} \cup \{4l + 15\}$ is difference-dominated, and $A_{2l+1} = A_{2l} \cup \{-4l - 1\}$ is MSTD.

Non-filling Method 3 Example

Table: Non-Filling in Method 3 Example Sequence

Set	$A_i + A_i$	$A_i - A_i$	Cardinality	Diameter	Density
A_1	32	31	10	16	0.625
A_2	36	37	11	20	0.550
A_3	40	39	12	24	0.500
A_4	44	45	13	28	0.464
A_5	48	47	14	32	0.437
A_6	52	53	15	36	0.416
A_7	56	55	16	40	0.400
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.250

Non-filling Method 3 Extension

We extend the general idea behind Non-filling Method 3 to create an even more efficient method.

Non-filling Method 3 Extension

We extend the general idea behind Non-filling Method 3 to create an even more efficient method.

- For $k \geq 0$, let

$$\begin{aligned} A_{4k+1} &= (5 \cdot [-k-5, k+6] + \{1,2\}) \\ &\quad \cup \{-7, 0, 5, 8, 15\} \setminus \{-9, 1, 2, 6, 7, 17\} \\ A_{4k+2} &= A_{4k+1} \cup \{5k+36\} \\ A_{4k+3} &= A_{4k+1} \cup \{-5k-28, 5k+36\} \\ A_{4k+4} &= A_{4k+1} \cup \{-5k-28, 5k+36, 5k+37\} \end{aligned}$$

Non-Filling Method 3 Extension Table

Table: Non-Filling in Method 3 Sequence

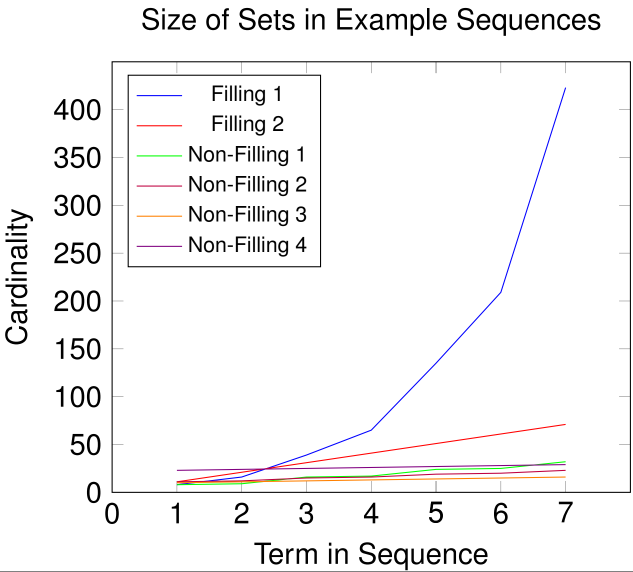
Set	$A_i + A_i$	$A_i - A_i$	Cardinality	Diameter	Density
A_1	98	97	23	56	0.410
A_2	102	103	24	60	0.400
A_3	106	105	25	64	0.391
A_4	110	111	26	65	0.400
A_5	114	113	27	66	0.409
A_6	118	119	28	70	0.400
A_7	122	121	29	74	0.392
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Limiting MSTD density: 0.400

Table: Comparison of MSTD set growth characteristics

Method	$\|A_1\|$	A_1 Diam.	Growth
Filling 1	≥ 8	≥ 14	Exponential
Filling 2	≥ 10	≥ 17	Linear
Non-Filling 1	≥ 8	≥ 14	Linear
Non-Filling 2	≥ 8	≥ 14	Linear
Non-Filling 3	≥ 11	≥ 18	Linear
Non-Filling 4	≥ 13	≥ 33	Linear

Growth Rates



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