MAKO Puzzles, 2024
Solutions
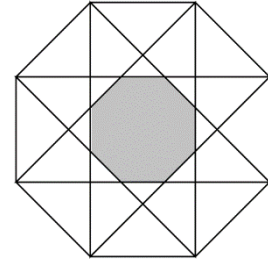
A perfect cube

Find the smallest perfect cube that ends with the digits 2024.

$674^3 = 306182024$

What is the ratio of the areas?

Find the ratio of the area of the larger regular octagon to that of the smaller (shaded) one in the figure shown.

$3 + 2\sqrt{2}$

Non-attacking bishops

Chess is played on an 8×8 board. A bishop is a piece that can move any of number of squares along a diagonal. What is the maximum number of bishops that can be placed on the board so that no bishop can attack another?

14 bishops

Find the limit

Evaluate the following limit.

$$\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}\cdots+\frac{1}{2n}\right)$$

ln 2

Find the roots of the polynomial

This problem is from the Missouri MAA Collegiate Competition.

A polynomial $P(x)$ is known to be of the form $P(x) = x^{15} - 9x^{14} + \cdots - 7$, where the ellipsis $(\cdots)$ represent unknown intermediate terms. It is also known that all roots of $P(x)$ are integers. Find the roots (including multiplicities) of $P(x)$.

7 is a single root, 1 is a root of multiplicity 8, and − 1 is a root of multiplicity 6