## MAKO Puzzles

## A Perfect Square

Find the smallest perfect square that begins with the digits 2019.
$2019241=\mathbf{1 4 2 1}^{\mathbf{2}}$

## Sangaku

Between the seventeenth and nineteenth centuries, Japan was completely isolated from the West. During that period, the Japanese developed their own unique style of mathematics. People from all walks of life would inscribe geometry problems on wooden tablets called sangaku and hang them in temples throughout Japan.


Here is a sangaku problem: Two quarter-circles and a semi-circle are inscribed in a unit square as shown in the figure above. The smaller circle is tangent to the two quarter-circles and to the side of the square. The larger circle is tangent to the two quarter-circles and to the semi-circle. Find the ratio of the radius of the larger circle to that of the smaller one.

The ratio is $8: 3$

## A Powerful Puzzler

Find the smallest positive integer such that half of it is a perfect square, a third of it is a perfect cube, and a fifth of it is a perfect fifth power.
$2^{15} 3^{10} 5^{6}$

## Minimize the Area

This problem is from the Missouri MAA Collegiate Competition.
Let $P$ be a point in the first quadrant lying on the parabola $y=x^{2}$ [other than the origin]. The normal line (the line perpendicular to the tangent line) to the parabola at $P$ will intersect the parabola a another point, say $Q$. Find the coordinates of $P$ so that the area bounded by the normal line and the parabola is a minimum.

$P=(1 / 2,1 / 4), Q=(-3 / 2,9 / 4)$, and the minimum area is $4 / 3$.

## Rules for playing KENKEN® ${ }^{\circledR}$

1) Choose a grid size.
2) Fill in the numbers from 1 to grid size.
3) Do not repeat a number in any row or column.
4) The numbers in each heavily outlined set of squares, called cages, must combine (in any order) to produce the target number in the top corner using the mathematical operation indicated.
5) Cages with just one square should be filled in with the target number in the top corner.
6) A number can be repeated within a cage as long as it is not in the same row or column.

| $2 \div$ | $6 \times$ | 1 | $7+$ |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
| $2-$ | $1-$ | $8+$ | 1 |
|  |  |  |  |


| $2-$ |  | $24 \times$ |  |  | $3-$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $9+$ |  |  | $6 \times$ |  |  |
| $10 \times$ | $3-$ | $60 \times$ |  | $4-$ |  |
|  |  | 2 |  | $24 \times$ |  |
| 6 | $16+$ |  | $2 \div$ |  | $1-$ |
|  |  |  |  |  |  |

