## MATH 103 - Sample Final Exam Review

This review is a collection of sample questions used by instructors of this course at Missouri State University. It contains a sampling of problems representing the material covered throughout the semester and may not contain every type of question on the final exam. Any material listed on the lecture schedule and/or the assignment sheet may be on the final exam. Please also be aware that a few questions on the final exam, while requiring knowledge and understanding of the content covered in the course, may be presented in a form different than the problems in the text.

Problems 1-12. Evaluate. Assume all variables are nonzero.

1. $\left(\frac{3}{2}\right)^{-3}$
2. $-8^{0}+7^{2}$
3. $-(-2)^{-2}+(2)^{-3}$
4. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$
5. $7^{\frac{4}{5}} \cdot 7^{\frac{6}{5}}$
6. $-\sqrt{81}$
7. $-125^{\frac{2}{3}}$
8. $\frac{-3}{4^{-2}}$
9. $\left(-9 x y^{3}\right)^{0}$
10. $(-32)^{\frac{2}{5}}$
11. $\left(\frac{1}{16}\right)^{\frac{1}{4}}$
12. $49^{-\frac{3}{2}}$

Problems 13-23. Simplify. Write answers with only positive exponents. Assume all variables are positive.
13. $y^{\frac{3}{2}} \cdot y^{\frac{2}{3}}$
14. $-3 x^{-3}$
15. $\left(\frac{x^{2}}{x^{6}}\right)^{-\frac{1}{2}}$
16. $\frac{-4 w^{-\frac{1}{3}}}{w^{\frac{5}{3}}}$
17. $\left(x^{2} y^{-4}\right)\left(\frac{x^{3}}{y}\right)^{2}$
18. $\left(\frac{8 x^{3}}{y^{6}}\right)^{\frac{2}{3}}$
19. $\left(4 m^{-3}\right)^{-2}\left(m^{4}\right)^{-1}$
20. $\left(5 a^{2} c^{-\frac{1}{2}} d^{\frac{1}{2}}\right)^{2}$
21. $\frac{x^{\frac{5}{4}} \cdot x^{-2}}{x^{\frac{3}{4}}}$
22. $\frac{-4 a^{5}\left(a^{-1}\right)^{3}}{\left(a^{-2}\right)^{-2}}$
23. $\left(\frac{-2 x^{6} y^{8}}{x^{-2} y^{10}}\right)^{-3}$

Problems 24-28. Simplify. Assume all variables are positive.
24. $\sqrt{-75}$
25. $-\sqrt{-81}$
26. $\sqrt{72 a^{3} b^{9} c^{17}}$
27. $\sqrt[3]{-135}$
28. $\sqrt[3]{27 x^{6} y^{11}}$

Problems 29-40. Factor completely. If the expression cannot be factored, write "prime".
29. $x^{2}-10 x+24$
30. $6 x^{2}+7 x-3$
31. $12 y^{2}+y-20$
32. $16 p^{2}-25$
33. $125 x^{3}-8$
34. $p^{3}+10 p^{2}+25 p$
35. $3 m^{2}-12 m-60$
36. $x^{2}+81$
37. $12 a x+8 a y-15 x-10 y$
38. $m^{3}+27 n^{3}$
39. $x^{3}-3 x^{2}+2 x-6$
40. $x^{4}+5 x^{2}-6$

Problems 41-61. Perform the indicated operation and simplify.
41. $(7 y-3)(4 y+5)$
42. $(4 z-3)(4 z+3)$
43. $3 x+(x-2)-(2 x+5)$
44. $\frac{(3 x-4)^{2}}{3 x^{2}} \cdot \frac{15 x}{9 x^{2}-16}$
45. $\frac{3 a-1}{a^{2}-9}-\frac{2}{a-3}$
46. $\frac{x^{2}-x-6}{x^{2}-8 x+15} \div \frac{x^{2}+5 x+6}{x^{2}-4 x-21}$
47. $\frac{x^{3}+64}{x^{2}-16} \div \frac{x^{2}-4 x+16}{x^{2}-4 x}$
48. $\frac{12}{x}-\frac{5}{4 x}$
49. $\frac{2}{x-3}+\frac{x}{x+3}$
50. $(6 x-5)^{2}$
51. $(4 x-3)\left(5 x^{2}+3 x-4\right)$
52. $-x y^{4}\left(5 x^{2} y-4 x y^{2}+3 x^{2} y^{2}\right)$
53. $\frac{2}{m^{2}-4 m+4}+\frac{3}{m^{2}+m-6}$
54. $\frac{2}{a-2}-\frac{a+2}{a^{2}-a-2}$
55. $\frac{x+2 y}{3 x}+\frac{3 y-x}{5 x}+2$
56. $\frac{2 p^{2}-7 p-4}{6 p^{2}+7 p+2}$
57. $\frac{6}{5 a+10}+\frac{7}{6 a+12}$
58. $\frac{m^{2}+2 m n-m-2 n}{m^{3}-1}$
59. $\frac{y^{2}-49}{y+2} \cdot \frac{y}{7-y}$
60. $\frac{5 x+6}{x^{2}+x-20}-\frac{-4+3 x}{x^{2}+x-20}$
61. $(x-3)+\left(2 x^{2}-2 x-3\right)$

Problems 62-86. Solve.
62. $5(a-3)-4(a+5)=-33$
63. $|x+2|=3$
64. $x^{2}-7 x+6=0$
65. $-2 x^{3}+50 x=0$
66. $|2 x+3|-3=4$
67. $(x-7)(x+5)=-20$
68. $3[3 m-2(m-3)]+4 m=31-7(1-2 m)$
69. $\frac{6 y}{5 y+15}-\frac{y+2}{3 y+9}=\frac{16 y-1}{15 y+45}$
70. $1+\frac{2}{x}=\frac{24}{x^{2}}$
71. $\frac{3}{x+1}-\frac{5}{x}=\frac{19}{x^{2}+x}$
72. $\sqrt{2 y-3}=4$
73. $|3 x-5|=|2 x+1|$
74. $(2 x-5)^{2}=49$
75. $2 x^{2}-4 x-5=0$
76. $\sqrt{3 x+13}-3=x$
77. $y^{4}+6 y^{2}-7=0$
78. $\frac{2 q+1}{3}-\frac{q-1}{4}=-2$
79. $\sqrt{z^{2}+5}=z+3$
80. $\frac{3}{x^{2}-25}=\frac{1}{x+5}+\frac{2}{x-5}$
81. $\left|\frac{x+5}{2}\right|+3=18$
82. $6 t^{2}=7-19 t$
83. $p=\sqrt{p^{2}+3 p+18}$
84. $(5 k-2)^{2}+7=31$
85. $\frac{4}{z+2}-\frac{1}{3 z+6}=\frac{11}{9}$
86. $4-3(m+2)+8 m=5(m-1)-7 m$

Problems 87 - 88. Solve by substitution.
87. $(x-1)^{2}-3(x-1)=28$
88. $3(x+3)^{2}-10(x+3)-8=0$

## Problems 89-90. Solve by completing the square.

89. $2 x^{2}+20 x+48=0$
90. $x^{2}-6 x=-14$

## Problems 91-97. Solve for the indicated variable.

91. $4 p-3(y+p)=2 y$ for $y$
92. $p=p x+y$ for $p$
93. $4 x-5 y=15$ for $x$
94. $a=\frac{3}{4}(b-5) \quad$ for $b$
95. $\frac{I}{r+R}=E \quad$ for $R$
96. $\frac{m-x}{n}=y$ for $m$
97. $9 x=3 y+b x+2$ for $x$

Problems 98-107. Solve and graph on a numberline. Write your solution in interval notation.
98. $\frac{3 x-2}{-2}<13$
99. $-\frac{2}{3} x+5 \geq 29$
100. $-2 \leq \frac{8-x}{3}<4$
101. $|2 n-1| \leq 4$
102. $|2 x-7|+11<3$
103. $|3 n-2|+9 \leq 26$
104. $|5 x+1|+2>8$
105. $|3-8 x|-5>2$
106. $|x-1| \geq 0$
107. $-4-(2+3 m) \leq 5 m+3$
108. Use the formula for slope to find the slope of the line through the points $(-3,2)$ and $(2,3)$.
109. Given the line $18=2 x$, find the slope and describe the line.
110. Find the $x$-intercept and $y$-intercept of $3 x-4 y=7$. Write them as ordered pairs.
111. Write $5 x+6 y=12$ in slope-intercept form, then state the slope and $y$-intercept.
112. Given $f(x)=5+3 x$ and $g(x)=-x^{2}-5$, find the following:
a. $\quad f(7)$
b. $g(7)$
c. $g(-2)$

Problems 113-115 Determine whether the pairs of lines are parallel or perpendicular to each other, or neither.
$x=6 y-3$
114.
$-y=3 x-2$

$$
y=5
$$

113. $3 x+\frac{1}{2} y=0$
$-6 x+2 y=6$
114. $y=-\frac{1}{5}$

Problems 116-120. Write the equation of the line. When possible, write the equation in slope-intercept form.
116. Slope is -3 and passes through $(2,3)$
119. Horizontal line that passes through $(8,-2)$
117. Passes through $(-1,3)$ and $(-3,2)$
120. Undefined slope that passes through $(-3,-4)$
118. Slope is $-\frac{2}{5}$ and passes through $(0,8)$

Problems 121-125. Give the domain and range of each relation. Determine whether the relation is a function.
121. $\{(8,0),(5,4),(9,3),(3,8)\}$
122.

123.

124.

125.


Problems 126-129. Determine where each function is undefined.
126. $h(x)=\frac{x+3}{x^{2}}$
127. $g(x)=\frac{6-x}{x^{2}-7 x}$
128. $k(x)=\frac{x+3}{x^{2}+9}$
129. $f(x)=\frac{x}{2 x-6}$

Problems 130-133. For each parabola, identify a) the vertex b) the axis of symmetry c) determine whether the parabola is wider, narrower, or the same width as $y=x^{2}$.
130. $y=\frac{3}{4} x^{2}$
131. $y=x^{2}+7$
132. $y=-2(x-5)^{2}+1$
133. $y=-\frac{1}{2}(x-2)^{2}$

Problems 134-143. Graph each of the following. Label at least 3 key points on the graph.
134. $5 x-y=5$
135. $y=-\frac{2}{3} x+3$
136. $y=\frac{3}{4} x^{2}$
137. $x=2 y$
138. $y=x^{2}+7$
139. $x=4$
140. $y=-2(x-5)^{2}+1$
141. $y-1=0$
142. $y=-\frac{1}{2}(x-2)^{2}$
143. Line with slope $\frac{3}{4}$ and passes through $(0,-5)$

## Problems 144-160. Applications. Define a variable, write an appropriate equation using the variable, solve.

144. A bookstore sells a college algebra book for $\$ 90$. If the bookstore makes a profit of $25 \%$ on each sale, what does the bookstore pay the publisher for each book?
145. The perimeter of a garden is 56 meters. The length is 11 meters less than twice the width. Find the dimensions of the garden.
146. Ryan can paint a room in 8 hours when working alone. If Stephanie helps him, the total job takes 6 hours. How long would it take Stephanie if she worked alone?
147. A certain airplane flies 250 mph in still air. On a windy day, this plane flew 46 miles into the wind in the same amount of time that it flew 54 miles with a tailwind. Find the speed of the wind.
148. Suppose that Boyd's Hardware just announced a $20 \%$ decrease in the price of their snowblower. If one particular snowblower model sells for $\$ 459.99$ after the decrease, find the original price of the snowblower.
149. A west-bound jet leaves an airport traveling 600 miles an hour. At the same time, an east-bound plane departs at 350 miles an hour. In how many hours will the planes be 1900 miles apart?
150. Joey drives from city A to city B at an average speed of 72 mph . Once he arrives at city B, he immediately turns around and heads back to city A . On the trip back to city A from city B , he has car trouble and drives an average speed of 48 mph . If the total round trip takes Joey 5 hours, what is the distance between city A and city B?
151. The hypotenuse of a right triangle is 1 centimeter more than twice the length of the shorter leg, and the longer leg is 9 centimeters shorter than three times the length of the shorter leg. Find the length of the three sides of the triangle.
152. Bob can paint a room in 3 hours working alone. It takes Barbara 5 hours to paint the same room. How long would it take them to paint the room together?
153. Natalie can ride her bike 4 mph faster than her husband, Chuck. If Natalie can ride 48 miles in the same time that Chuck can ride 24 miles, what are their speeds?
154. When Sarah takes the bus to work, the trip takes 30 minutes. When she takes the train to work, the trip takes 20 minutes. The average speed of the train is 15 mph more than the average speed of the bus. Find the distance to work.
155. After $t$ seconds, the height $h(t)$ of a model rocket launched from the ground into the air is given by the function $h(t)=-16 t^{2}+80 t$. Find how long it takes the rocket to reach a height of 96 feet.
156. Mary put some money in a certificate of deposit earning $4.2 \%$ simple interest. She deposited twice that amount in a money market account paying $4 \%$ simple interest. After 1 year her total interest was $\$ 488$. How much did Mary deposit in her money market account?
157. Two cars leave towns 312 miles apart at the same time traveling toward each other. One car travels 12 mph faster than the other car. They meet 3 hours later. What is the speed of the faster car?
158. How many liters of a $10 \%$ alcohol solution should be mixed with 12 liters of a $20 \%$ alcohol solution to obtain a $14 \%$ alcohol solution?
159. A coffee merchant combines coffee A costing $\$ 6$ per pound with coffee B costing $\$ 3.50$ per pound. How many pounds of each should be used to make 25 pounds of a blend costing $\$ 5.25$ per pound?
160. The proportion of males to females at a certain college is $3: 5$. If the total population of the college is 1960 students, how many of these are female?

## SOLUTIONS

1. $\frac{8}{27}$
2. 48
3. $-\frac{1}{8}$
4. $\frac{25}{4}$
5. 49
6. -9
7. -25
8. -48
9. 1
10. 4
11. $\frac{1}{2}$
12. $\frac{1}{343}$
13. $y^{\frac{13}{6}}$
14. $-\frac{3}{x^{3}}$
15. $x^{2}$
16. $-\frac{4}{w^{2}}$
17. $\frac{x^{8}}{y^{6}}$
18. $\frac{4 x^{2}}{y^{4}}$
19. $\frac{m^{2}}{16}$
20. $\frac{25 a^{4} d}{c}$
21. $\frac{1}{x^{\frac{3}{2}}}$
22. $-\frac{4}{a^{2}}$
23. $-\frac{y^{6}}{8 x^{24}}$
24. $5 i \sqrt{3}$
25. $-9 i$
26. $6 a b^{4} c^{8} \sqrt{2 a b c}$
27. $-3 \sqrt[3]{5}$
28. $3 x^{2} y^{3} \sqrt[3]{y^{2}}$
29. $(x-6)(x-4)$
30. $(2 x+3)(3 x-1)$
31. $(4 y-5)(3 y+4)$
32. $(4 p+5)(4 p-5)$
33. $(5 x-2)\left(25 x^{2}+10 x+4\right)$
34. $p(p+5)^{2}$
35. $3\left(m^{2}-4 m-20\right)$
36. prime
37. $(3 x+2 y)(4 a-5)$
38. $(m+3 n)\left(m^{2}-3 m n+9 n^{2}\right)$
39. $\left(x^{2}+2\right)(x-3)$
40. $\left(x^{2}+6\right)(x+1)(x-1)$
41. $28 y^{2}+23 y-15$
42. $16 z^{2}-9$
43. $2 x-7$
44. $\frac{5(3 x-4)}{x(3 x+4)}$
45. $\frac{a-7}{(a+3)(a-3)}$
46. $\frac{x-7}{x-5}$
47. $x$
48. $\frac{43}{4 x}$
49. $\frac{x^{2}-x+6}{(x+3)(x-3)}$
50. $36 x^{2}-60 x+25$
51. $20 x^{3}-3 x^{2}-25 x+12$
52. $-5 x^{3} y^{5}+4 x^{2} y^{6}-3 x^{3} y^{6}$
53. $\frac{5 m}{(m-2)^{2}(m+3)}$
54. $\frac{a}{(a+1)(a-2)}$
55. $\frac{32 x+19 y}{15 x}$
56. $\frac{p-4}{3 p+2}$
57. $\frac{71}{30(a+2)}$
58. $\frac{m+2 n}{m^{2}+m+1}$
59. $-\frac{y(y+7)}{y+2}$
60. $\frac{2}{x-4}$
61. $2 x^{2}-x-6$
62. $a=2$
63. $x=-5,1$
64. $x=1,6$
65. $x=-5,0,5$
66. $x=-5,2$
67. $x=-3,5$
68. $m=-\frac{6}{7}$
69. no solution
70. $x=-6,4$
71. $x=-12$
72. $y=\frac{19}{2}$
73. $x=\frac{4}{5}, 6$
74. $x=-1,6$
75. $x=\frac{2+\sqrt{14}}{2}, \frac{2-\sqrt{14}}{2}$
76. $x=1$
77. $y=-1,1,-i \sqrt{7}, i \sqrt{7}$
78. $q=-\frac{31}{5}$
79. $z=-\frac{2}{3}$
80. $x=-\frac{2}{3}$
81. $x=-35,25$
82. $t=-\frac{7}{2}, \frac{1}{3}$
83. no solution
84. $k=\frac{2+2 \sqrt{6}}{5}, \frac{2-2 \sqrt{6}}{5}$
85. $z=1$
86. $m=-\frac{3}{7}$
87. $x=-3,8$
88. $x=-\frac{11}{3}, 1$
89. $x=-6,-4$
90. $x=3+i \sqrt{5}, 3-i \sqrt{5}$
91. $y=\frac{p}{5}$
92. $p=\frac{y}{1-x}$
93. $x=\frac{5 y+15}{4}$
94. $b=\frac{4 a+15}{3}$ or $b=\frac{4}{3} a+5$
95. $R=\frac{I-E r}{E}$ or $R=\frac{I}{E}-r$
96. $m=n y+x$
97. $x=\frac{3 y+2}{9-b}$
98. $(-8, \infty)$
99. $(-\infty,-36]$
100. $(-4,14]$
101. $\left[-\frac{3}{2}, \frac{5}{2}\right]$
102. no solution
103. $\left[-5, \frac{19}{3}\right]$
104. $\left(-\infty,-\frac{7}{5}\right) \cup(1, \infty)$
105. $\left(-\infty,-\frac{1}{2}\right) \cup\left(\frac{5}{4}, \infty\right)$
106. $(-\infty, \infty)$
107. $\left[-\frac{9}{8}, \infty\right)$
108. slope is $\frac{1}{5}$
109. The slope is undefined. vertical line passing through $(9,0)$
110. $x$-intercept is $\left(\frac{7}{3}, 0\right)$ $y$-intercept is $\left(0,-\frac{7}{4}\right)$
111. $y=-\frac{5}{6} x+2$
slope is $-\frac{5}{6}$
$y$-intercept is $(0,2)$
112. 

a) 26
b) -54
c) -9
113. perpendicular
114. neither
115. parallel
116. $y=-3 x+9$
117. $y=\frac{1}{2} x+\frac{7}{2}$
118. $y=-\frac{2}{5} x+8$
119. $y=-2$
120. $x=-3$
121. Domain $=\{3,5,8,9\}$

Range $=\{0,3,4,8\}$
Is a function
122. Domain $=(-\infty, \infty)$

Range $=[-4, \infty)$
Is a function
123. Domain $=\{-4,-3,-2,-1,0,1\}$

Range $=\{1,2\}$
Not a function
124. Domain $=[-5,8]$

Range $=[-2,4]$
Is a function
125. Domain $=[-6, \infty)$

Range $=(-4, \infty)$
Not a function
126. $x=0$
127. $x=0,7$
128. undefined nowhere
129. $x=3$
130. a) vertex is $(0,0)$
b) $x=0$
c) wider
131. a) vertex is $(0,7)$
b) $x=0$
c) same width
132. a) vertex is $(5,1)$
b) $x=5$
c) narrower
133. a) vertex is $(2,0)$
b) $x=2$
c) wider
134.

135.

136.

137.

138.

139.

140.

141.

142.

143.

144. $\$ 72$
145. length is 15 meters width is 13 meters
146. 24 hours
147. 20 mph
148. $\$ 574.99$
149. 2 hours
150. 144 miles
151. $8 \mathrm{~cm}, 15 \mathrm{~cm}, 17 \mathrm{~cm}$
152. $1 \frac{7}{8}$ hours $=1.875$ hours
153. Chuck's speed is 4 mph

Natalie's speed is 8 mph
154. 15 miles
155. 2 seconds
156. $\$ 8000$
157. 58 mph
158. 18 liters
159. $\quad 17.5 \mathrm{lbs}$ of coffee A 7.5 lbs of coffee B
160. 1225 female students

